Question 1: *Compactness Parameter of Active Galactic Nuclei*

a) The maximum possible luminosity for a spherically accreting compact object is given by balancing the gravitational force acting on the accreted protons against the radiation force acting on the accreted electrons,

$$F_{\mathrm{rad}} = \frac{\sigma_{\mathrm{T}} S}{c} \tag{w1.1}$$

where $S = L/4\pi r^2$. Show that this *Eddington luminosity* is given by

$$L_{\mathrm{Edd}} = \frac{4\pi G M m_{\mathrm{p}} c}{\sigma_{\mathrm{T}}} \tag{w1.2}$$

b) In the lectures the compactness parameter was introduced as

$$\ell = \frac{L\sigma_{\mathrm{T}}}{R m_{\mathrm{e}} c^3} \tag{s1.1}$$

Typical radii for compact objects are on the order of the *Schwarzschild radius*,

$$r_{\mathrm{S}} = \frac{2GM}{c^2} \tag{s1.2}$$

Show that

$$\ell = \frac{2\pi}{3}\frac{m_{\mathrm{p}}}{m_{\mathrm{e}}}\left(\frac{L}{L_{\mathrm{Edd}}}\right)\left(\frac{3r_{\mathrm{S}}}{R}\right) \tag{s1.3}$$

Since $L < L_{\mathrm{Edd}}$ and $R \sim r_{\mathrm{S}}$, for typical sources $\ell \lesssim 4000$.

c) The Compton power radiated by a higly relativistic electron through inverse Compton scattering was found in the lecture to be

$$P_{\mathrm{compt}} = \frac{4}{3}\sigma_{\mathrm{T}} c \gamma^2 \beta^2 U_{\mathrm{rad}} \tag{s1.1}$$

where $U_{\mathrm{rad}} = S/c$. Show that the time scale for energy loss due to the Compton loss of highly relativistic electrons with energy $E = \gamma m_{\mathrm{e}} c^2$ is given by

$$t_{\mathrm{Compt}} = \frac{E}{P} = \frac{1}{\gamma}\frac{3\pi}{\ell}\frac{R}{c} \tag{s1.2}$$

Since $3\pi \sim 10$, for $\ell > 10$ the Compton cooling time of highly energetic particles in the system is shorter than their escape time (since $t_{\mathrm{esc}} \sim R/c$) and we would expect typical accreting systems to have $\ell \sim 1$.