Large Scale Structures and Structure Formation

The Lumpy Universe

So far: treated universe as smooth universe.

In reality:

Universe contains structures!

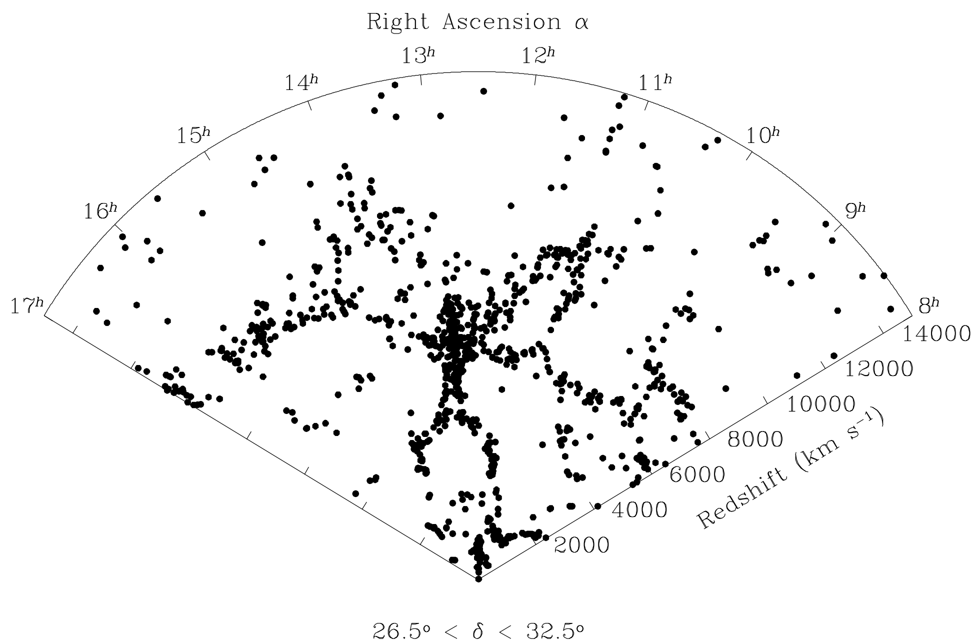
Last part of this class:

1. What are structures?
2. How can we quantify them?
3. How do structures form?
4. How do structures evolve?

Will see that all these questions are deeply connected with parameters of the universe seen so far:

1. H_0
2. Ω_0, Ω_b, Ω_m, Ω_Λ,...
3. Existence and Nature of Dark Matter

Introduction, I



(de Lapparent, Geller & Huchra, 1986, limiting mag $m_{\mathrm{B}} = 15.6$)

Lumpy universe: spatial distribution of galaxies and greater structures.

Observationally: need distance information for many (10^4) objects

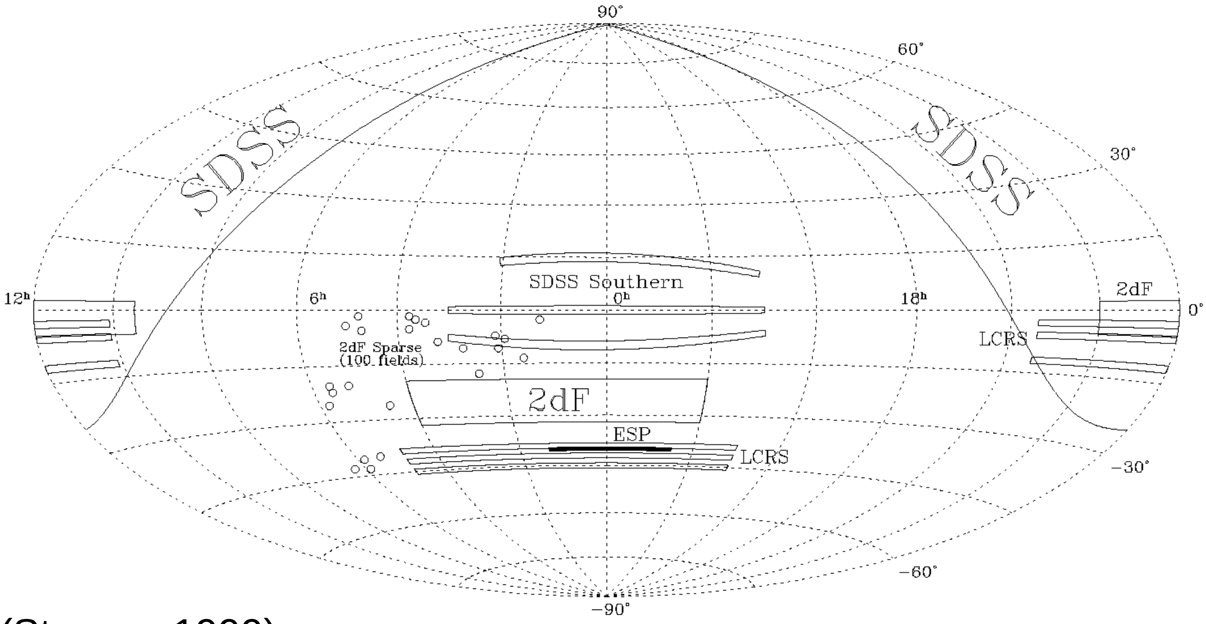
$\Longrightarrow$ Large redshift surveys

Review: Strauss & Willick (1995)

Redshift survey: Survey of (patch of) sky determining galaxy z and position to predefined magnitude or z.

First larger survey: de Lapparent, Geller & Huchra (1986)

Introduction, II



(Strauss, 1999)

Classification:

1D-surveys: very deep exposures of small patch of sky, e.g. HST Deep Field, Lockman Hole Survey, Marano Field.

2D-surveys: cover long strip of sky, e.g., CfA-Survey ($1.5 \times 100^\circ$), 2dF-Survey (“2 degree Field”).

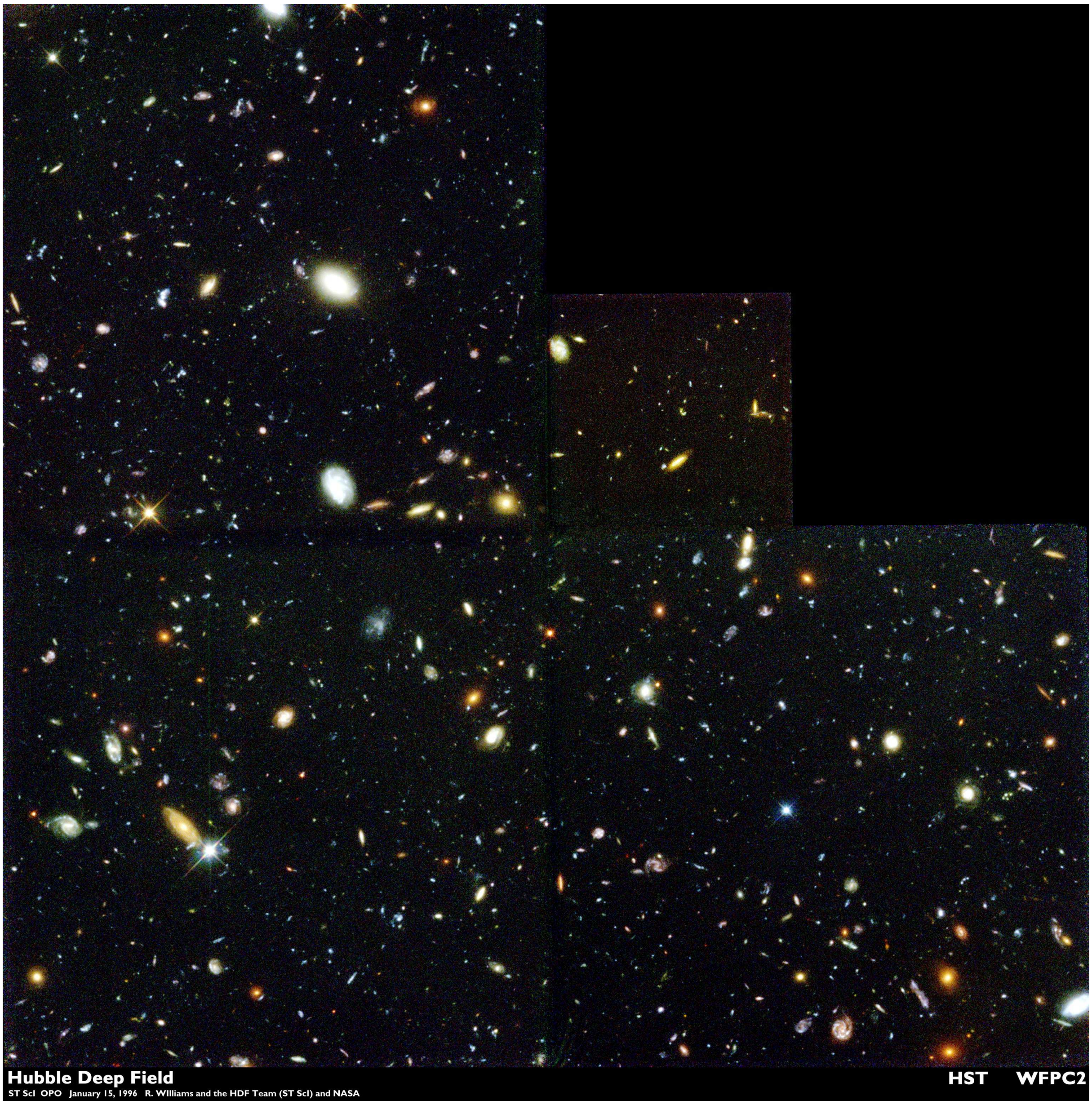
3D-surveys: cover part of the sky, e.g., Sloan Digital Sky Survey.

These surveys attempt to go to certain limit in z or m.

Other approaches: use pre-existing galaxy catalogues (e.g., QDOT Survey [IRAS galaxies], APM survey,...).

Will concentrate here on the larger surveys based on no other catalogue.

1D Surveys

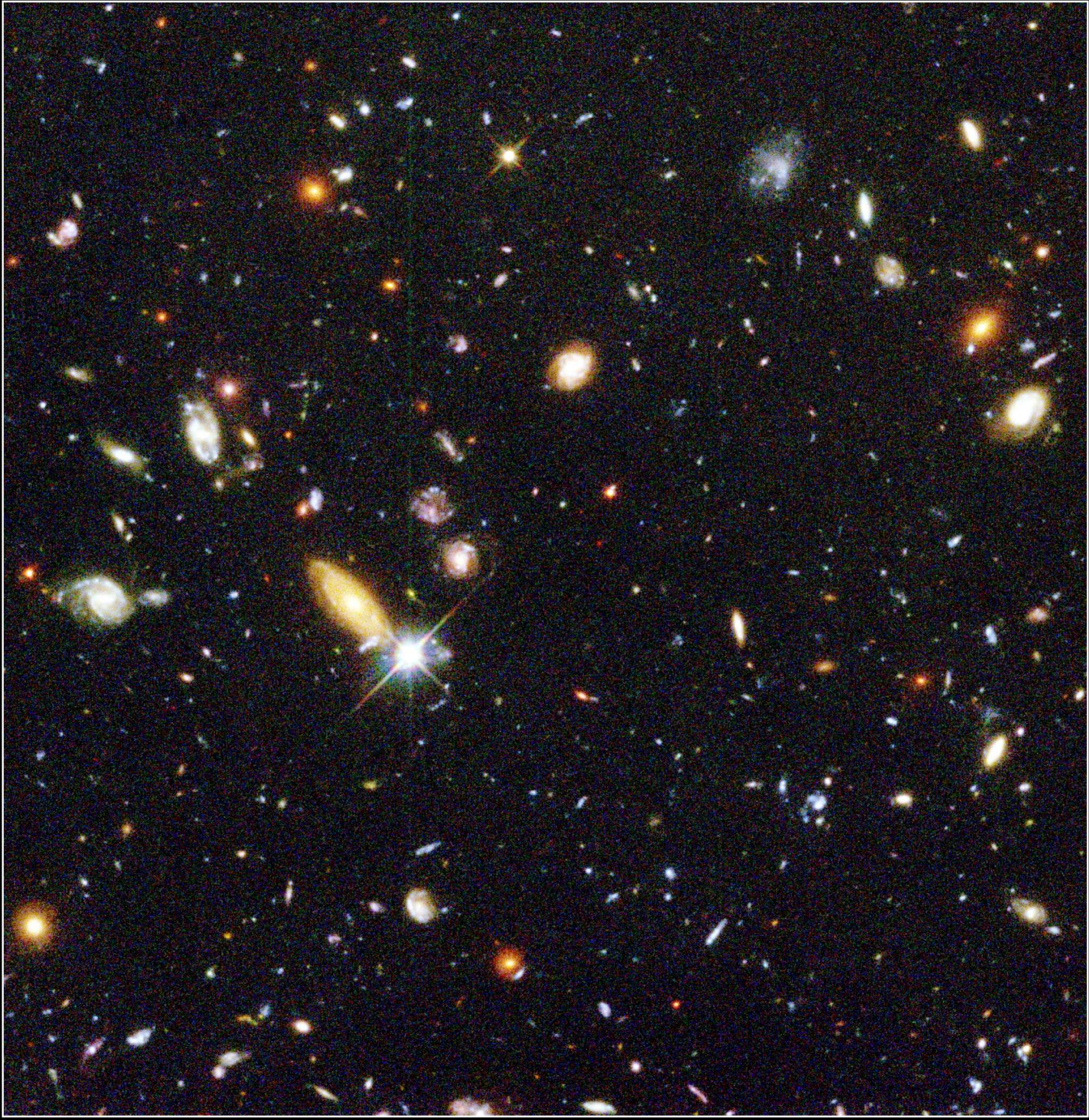


Hubble Deep Field, courtesy STScI

HDF: ~ 150 ksec/Filter for 4 HST Filters made in 1995 December.

Many galaxies with weird shapes $\Longrightarrow$ protogalaxies!

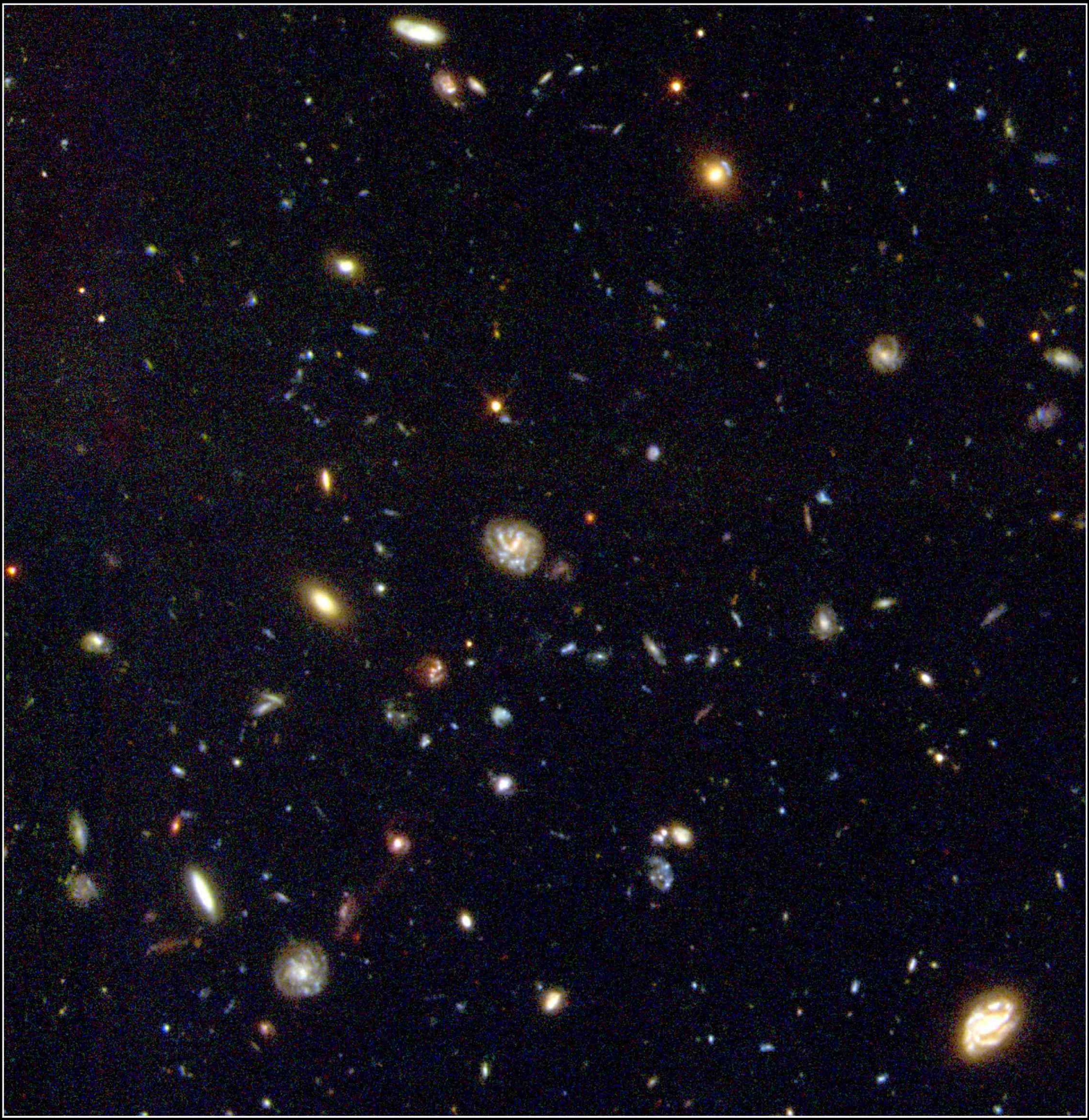
Redshifts: $z \in [0.5, 5.3]$ (Fernández-Soto et al., 1999)



Hubble Deep Field

Hubble Space Telescope • WFPC2

PRC96-01a • ST ScI OPO • January 15, 1995 • R. Williams (ST ScI), NASA

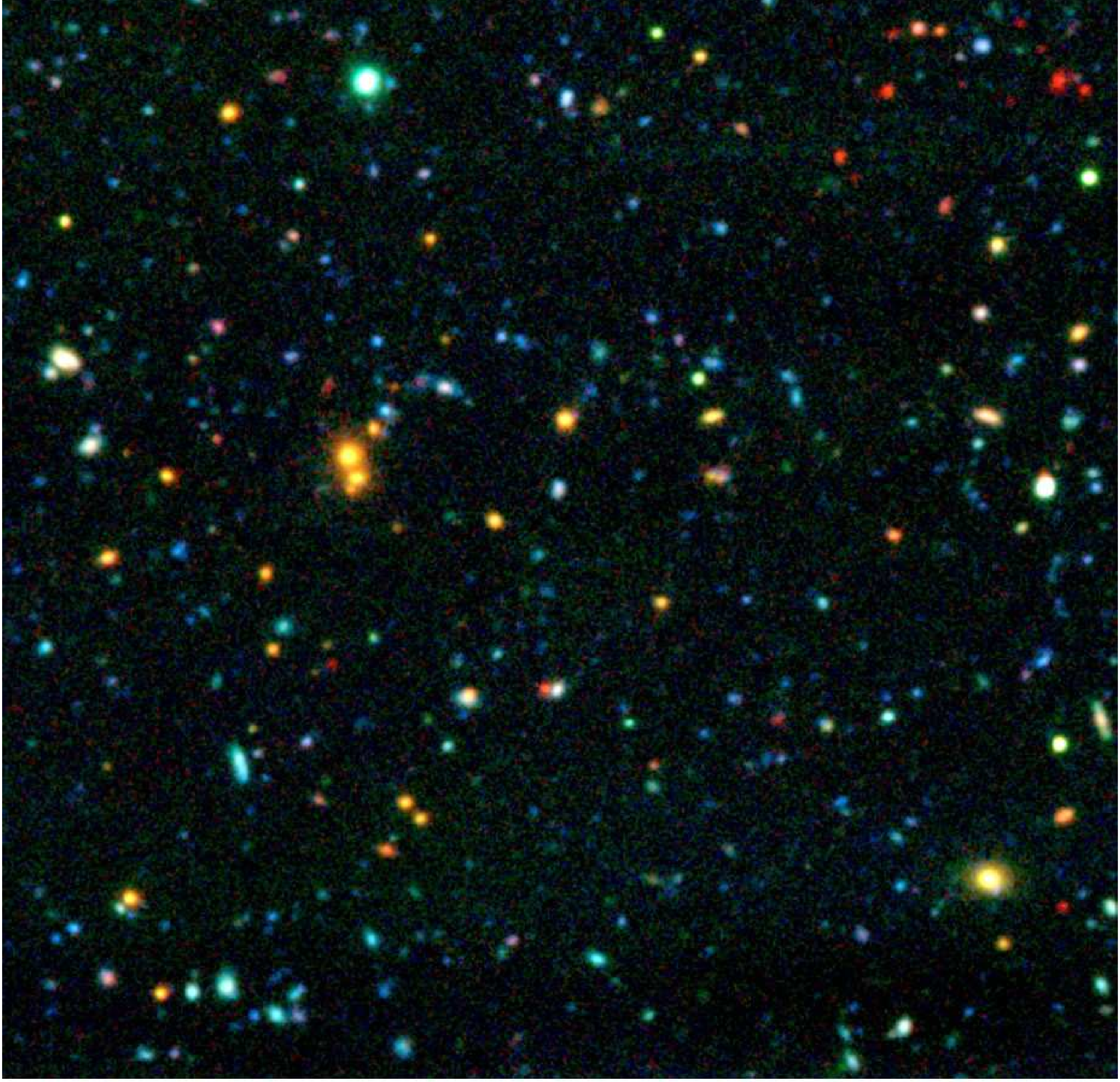


Hubble Deep Field South

Hubble Space Telescope • WFPC2

PRC98-41a • November 23, 1998 • STScI OPO • The HDF-S Team and NASA

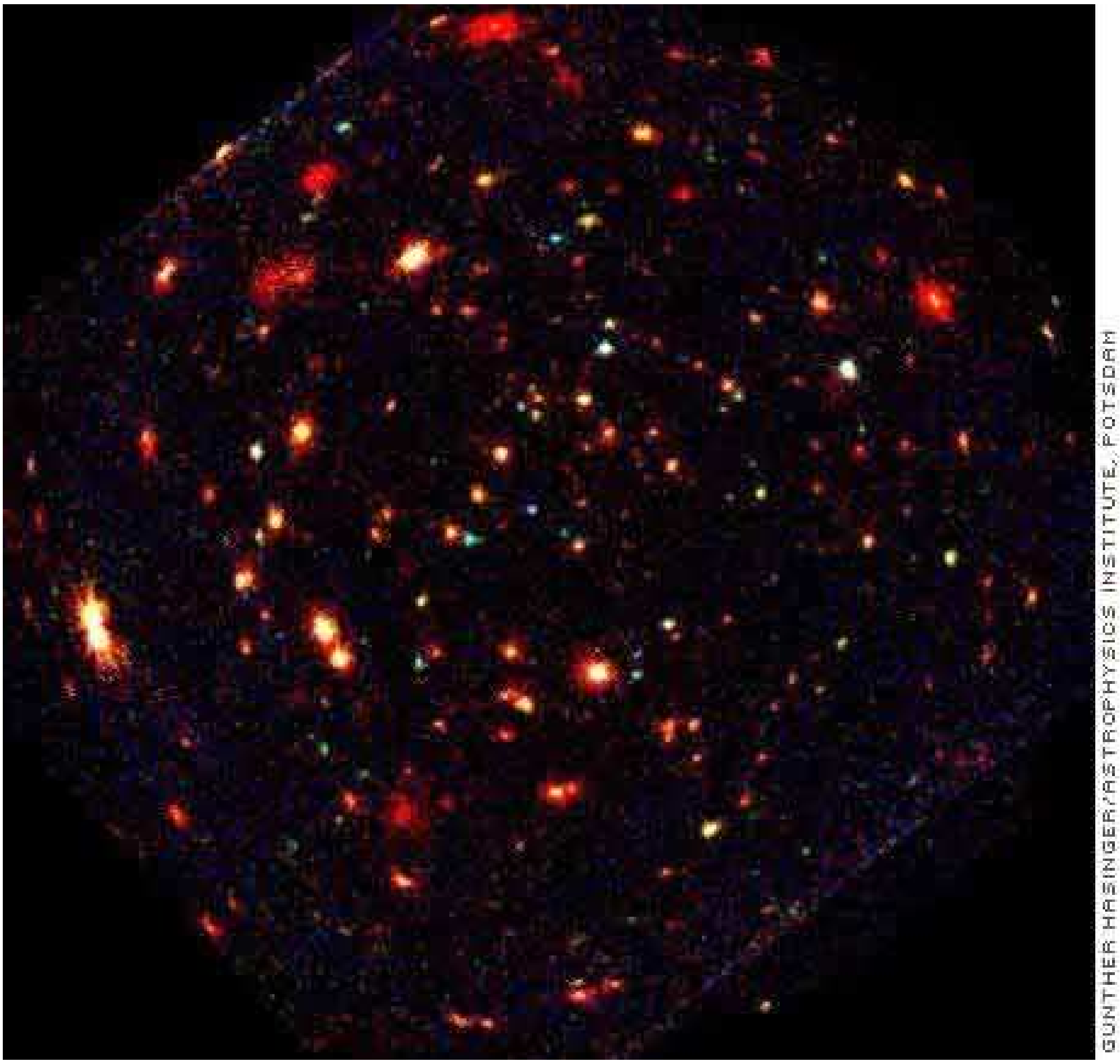
1998: Hubble Deep Field South, 10 d of total observing time!



Distant Galaxies in "AXAF Deep Field" (VLT ANTU / ISAAC + NTT / SUSI-2)

ESO PR Photo 06b/00 (17 February 2000)

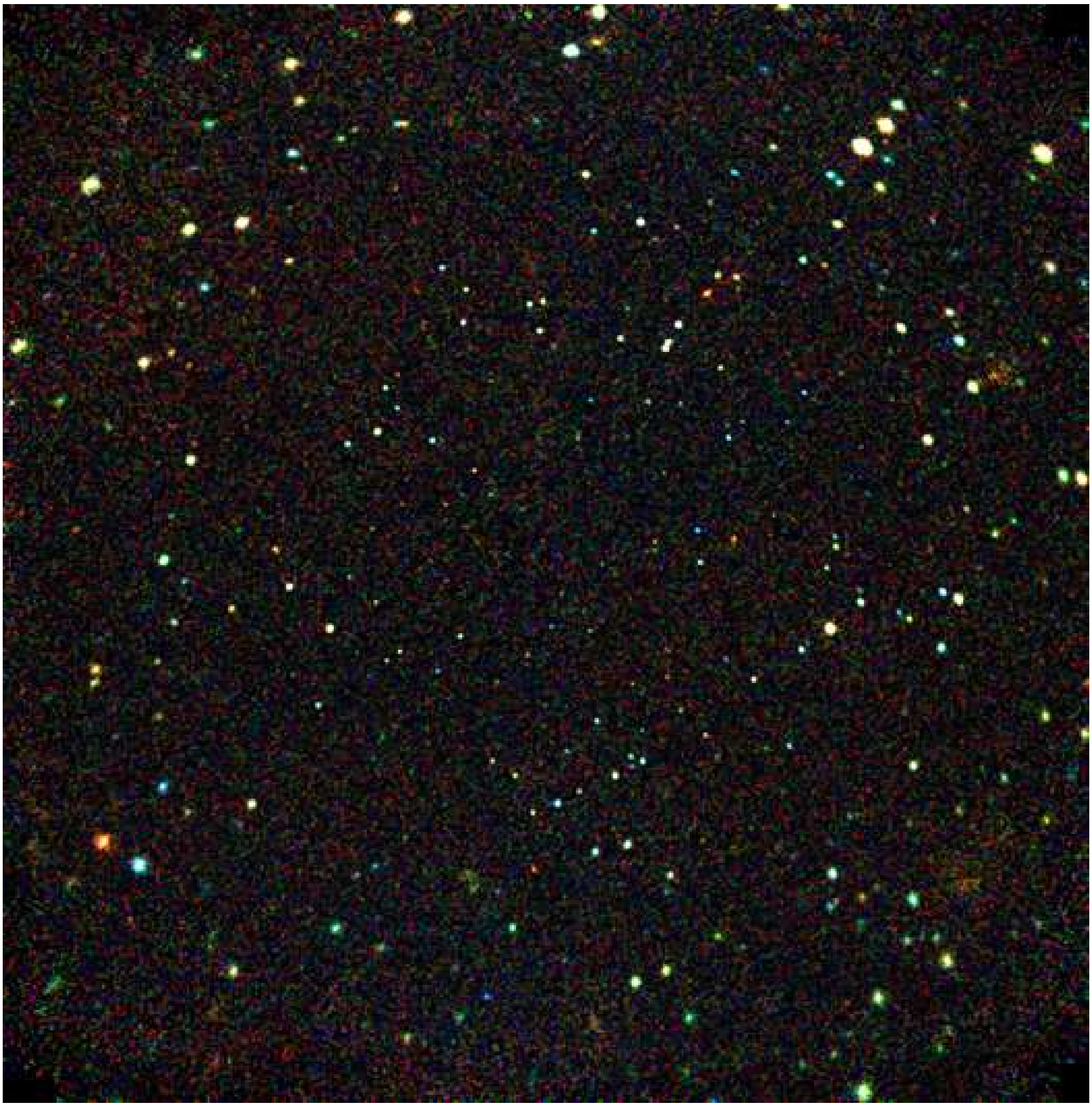
1D Surveys



XMM-Newton, Hasinger et al., 2001,
blue: hard X-ray spectrum,
red: soft X-ray spectrum

Lockman Hole: Northern Sky region with very low N_{H}
$\Longrightarrow$ low interstellar absorption
$\Longrightarrow$ “Window in the sky”
$\Longrightarrow$ X-rays: evolution of active galaxies with z!

1D Surveys

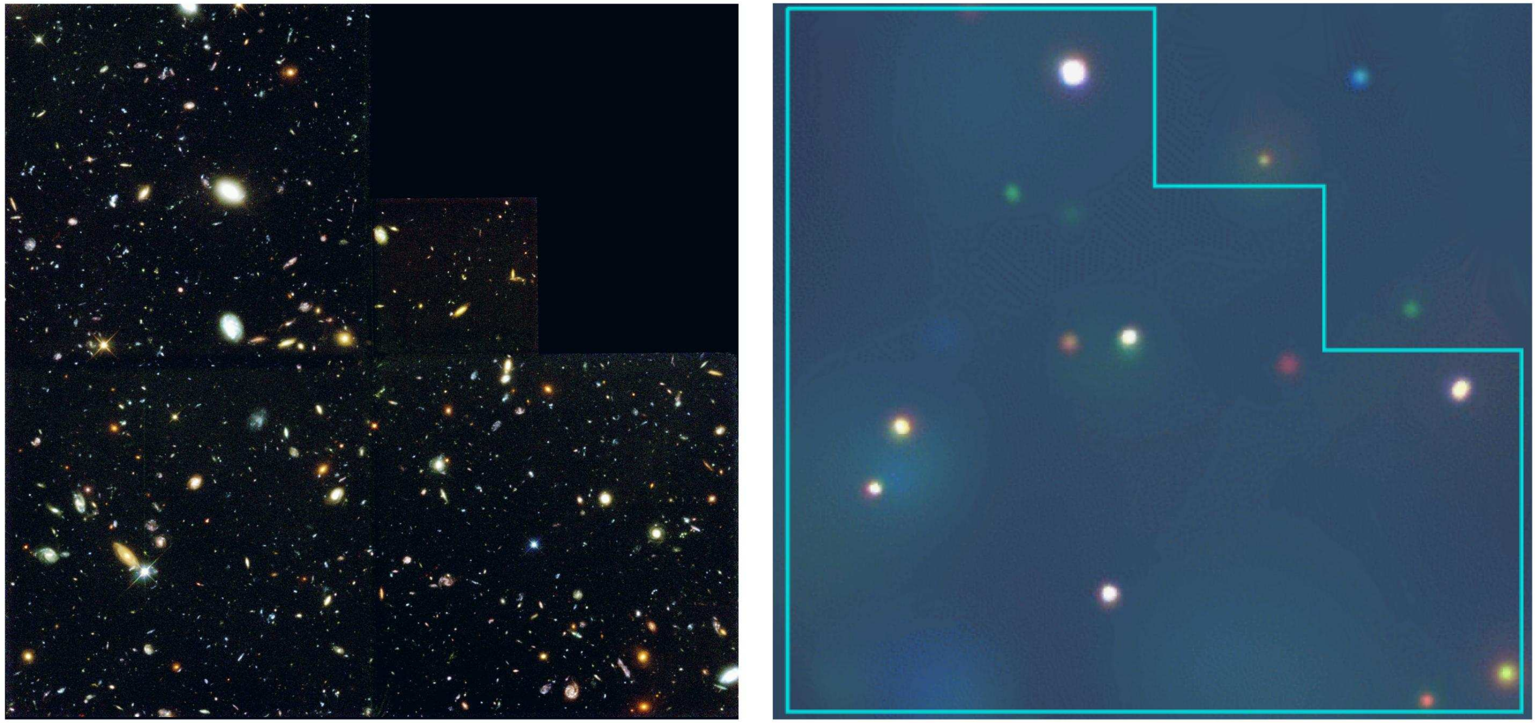


scale: 15′ × 15′; courtesy NASA/JHU/AUI/R.Giacconi et al.

Chandra Deep Field South: 1 Msec (10.8 days) on one region in Fornax $\Longrightarrow$ Deepest X-ray field ever...

color code: spectral hardness

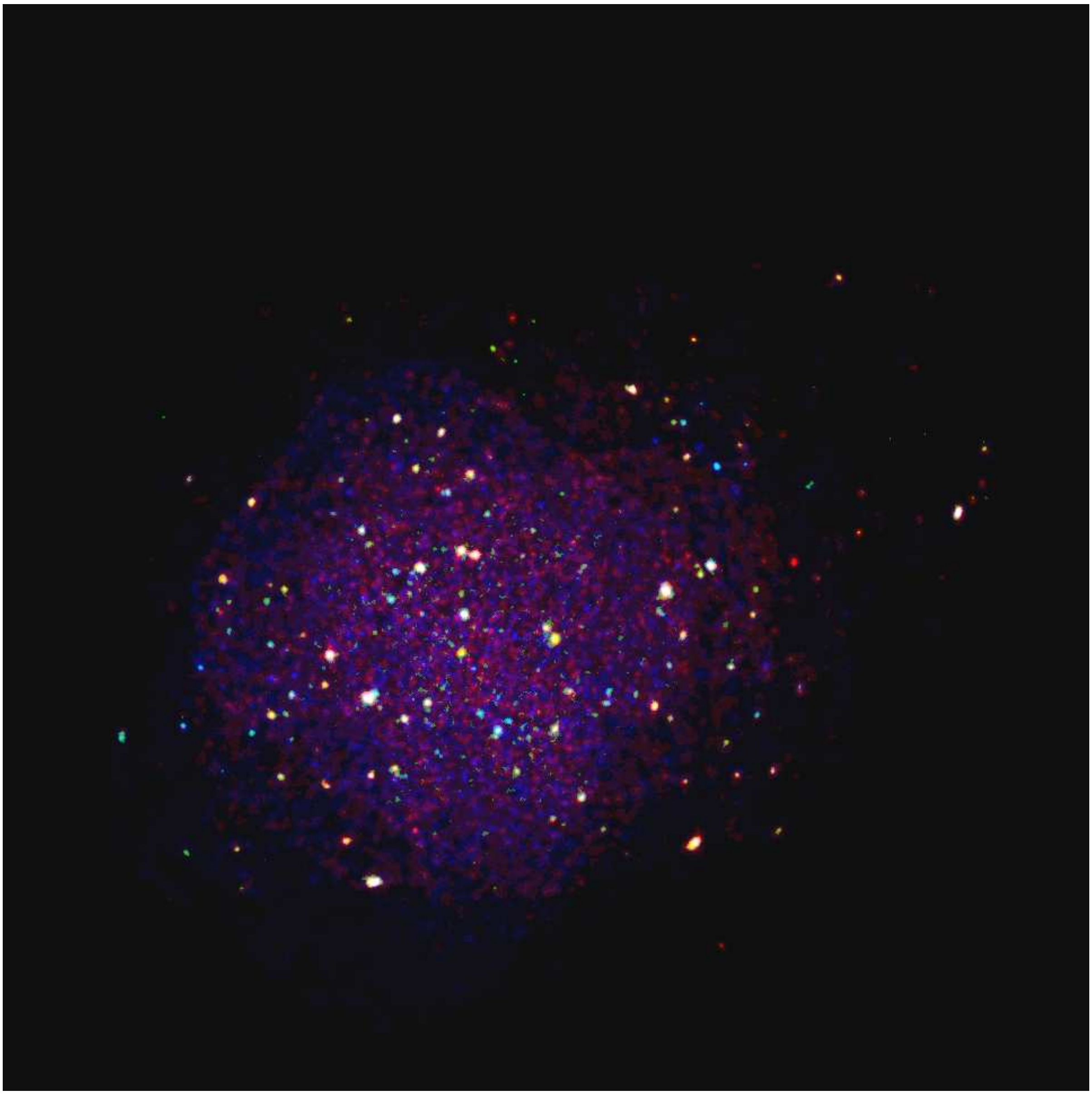
1D Surveys



Chandra/HST Image of Hubble Deep Field North; 500 ksec

Joint multi-wavelength campaigns allow the measurement of broad-band spectra of sources in the early universe!

1D Surveys



Deep *XMM-Newton* image of the Marano Field (IAAT/AIP/MPE)

1D Surveys ("Deep Exposures") give snapshot of evolution of galaxies over large z.

2D/3D Surveys: Technology

Future for Large Scale Structure: 2D and 3D Surveys observing large part of sky with dedicated instruments.

Currently largest surveys:

Las Campanas Redshift Survey (LCRS): 26418 redshifts in six $1.5 \times 80^\circ$ slices around NGP and SGP, out to $z = 0.2$.

CfA Redshift Survey: 30000 galaxies

APM: (Oxford University) $2 \sim 10^6$ galaxies, 10^7 stars around SGP, 10% of sky, through $B = 21$ mag.

2MASS: IR Survey of complete sky (Mt. Hopkins/CTIO) completed 2000 October 25), 3 bands, $\sim 2 \times 10^6$ galaxies, accompanying redshift survey (8dF, CfA)

Sloan Digital Sky Survey (SDSS): dedicated 2000 October 5, Apache Point Obs., NM, 25% of whole sky, $\sim 10^8$ objects,

And many more (e.g., Keck, ESO,...).

2D/3D Surveys: Technology

courtesy SDSS

SDSS 2.5 m telescope at Apache Point Observatory

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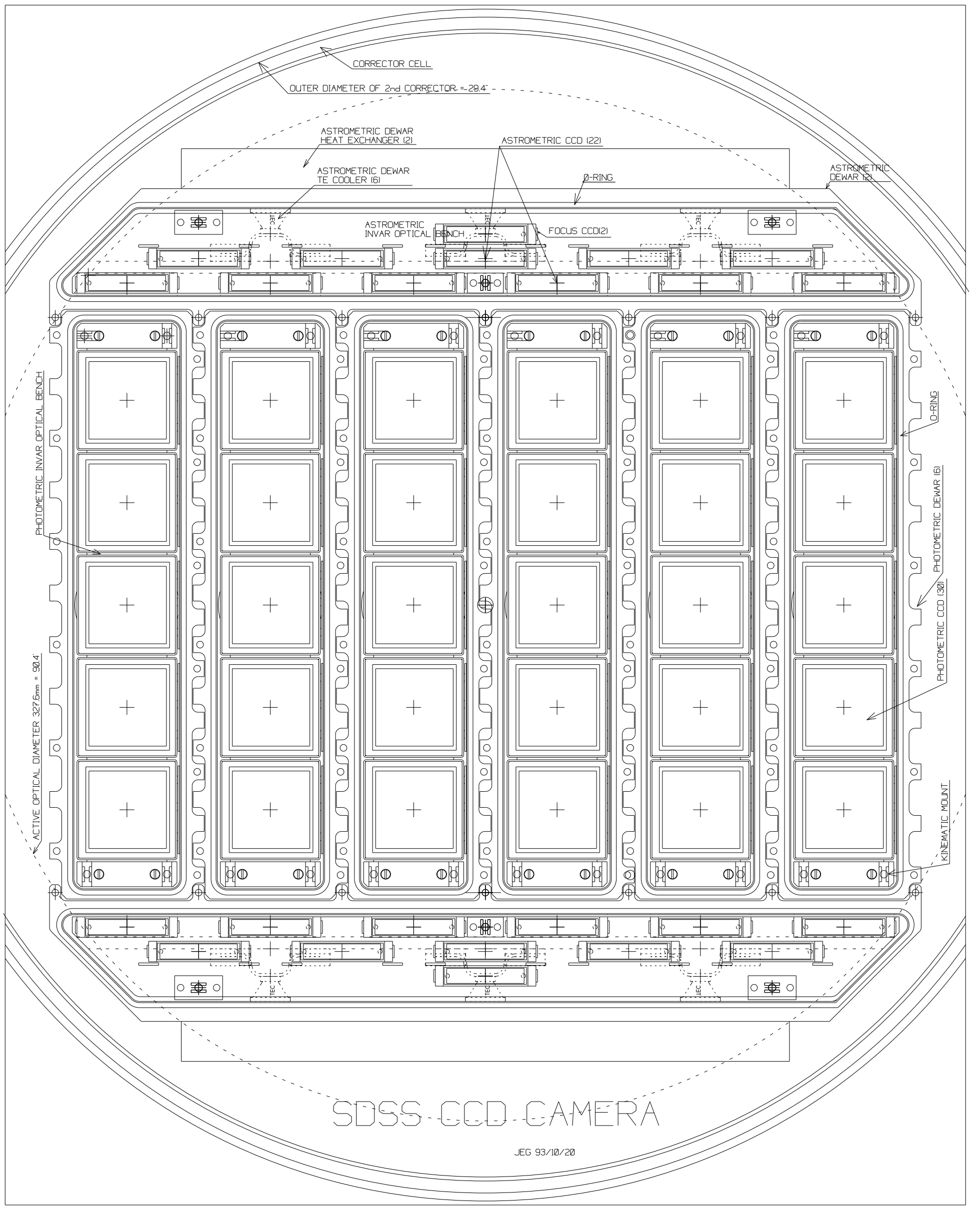
2D/3D Surveys: Technology



(Strauss, 1999, Fig. 5)

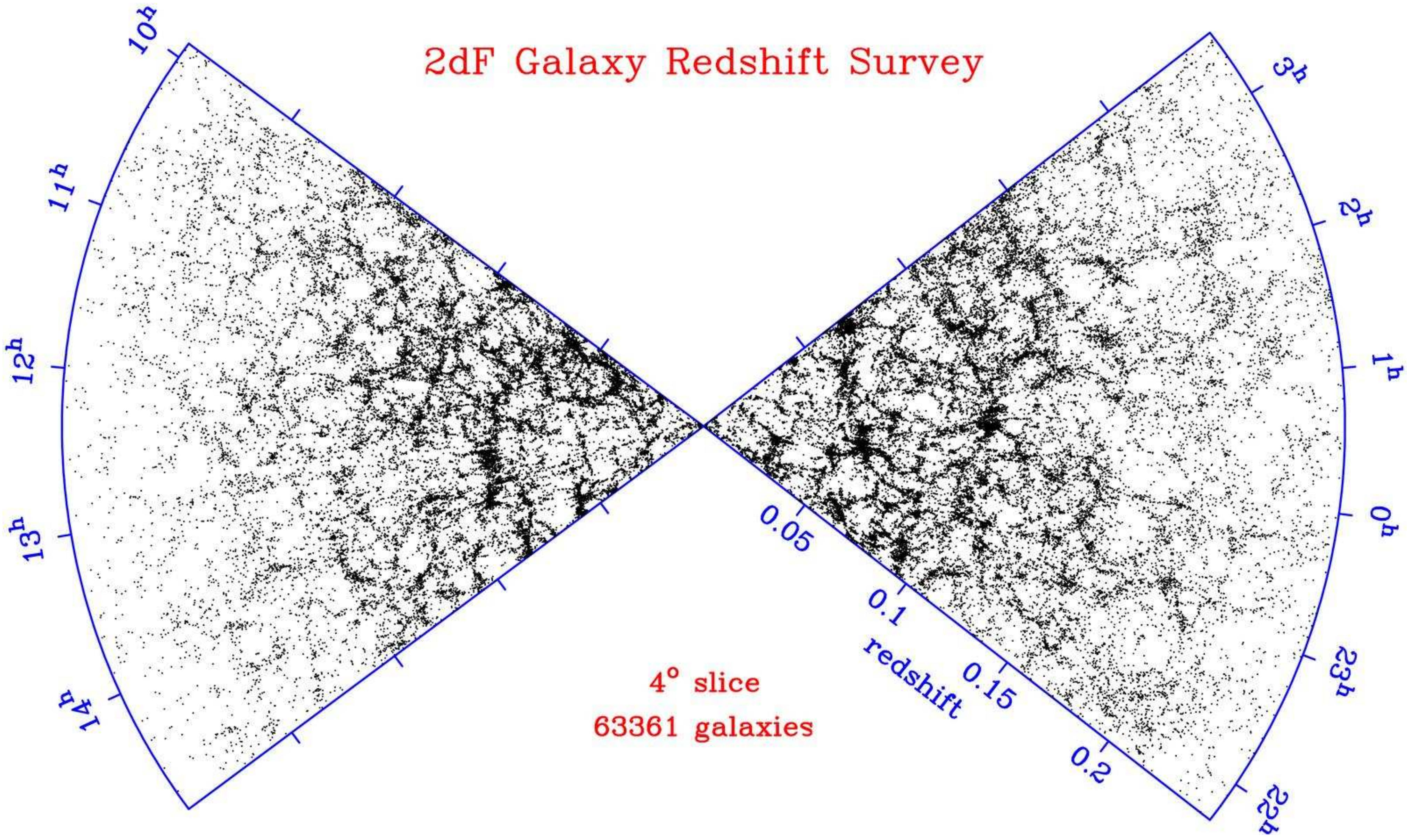
CCD alignment of SDSS:

- focal plane: 2.5°,
- 5 rows of 2048 $\times$ 2048 CCDs with r, i, u, z, g filters, saturation at $r = 14$
- 22 2048 $\times$ 400 CCD, saturation at $r = 6.6$ for astrometry

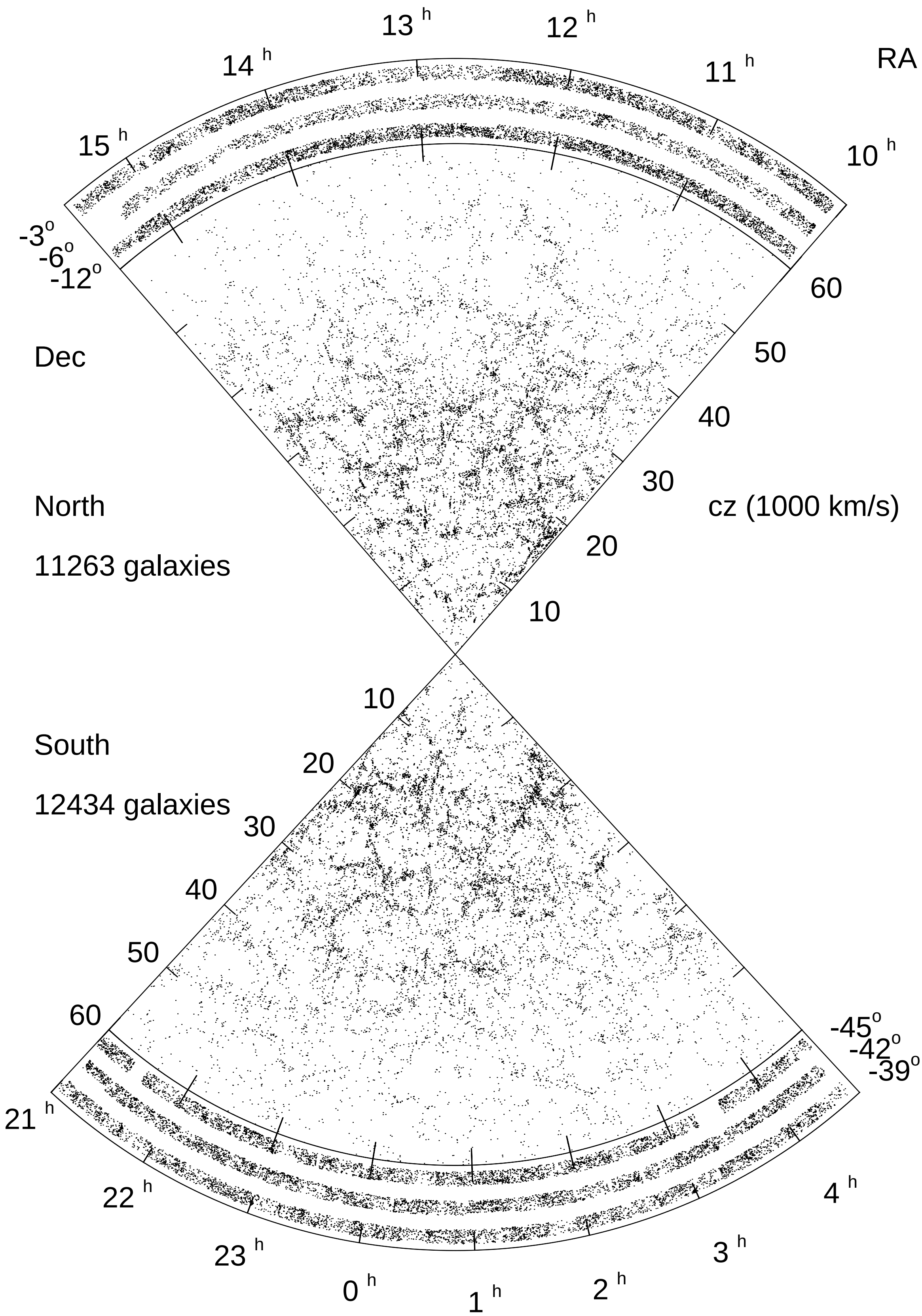
Imaging by slewing over CCD Array

2D/3D Surveys: Technology

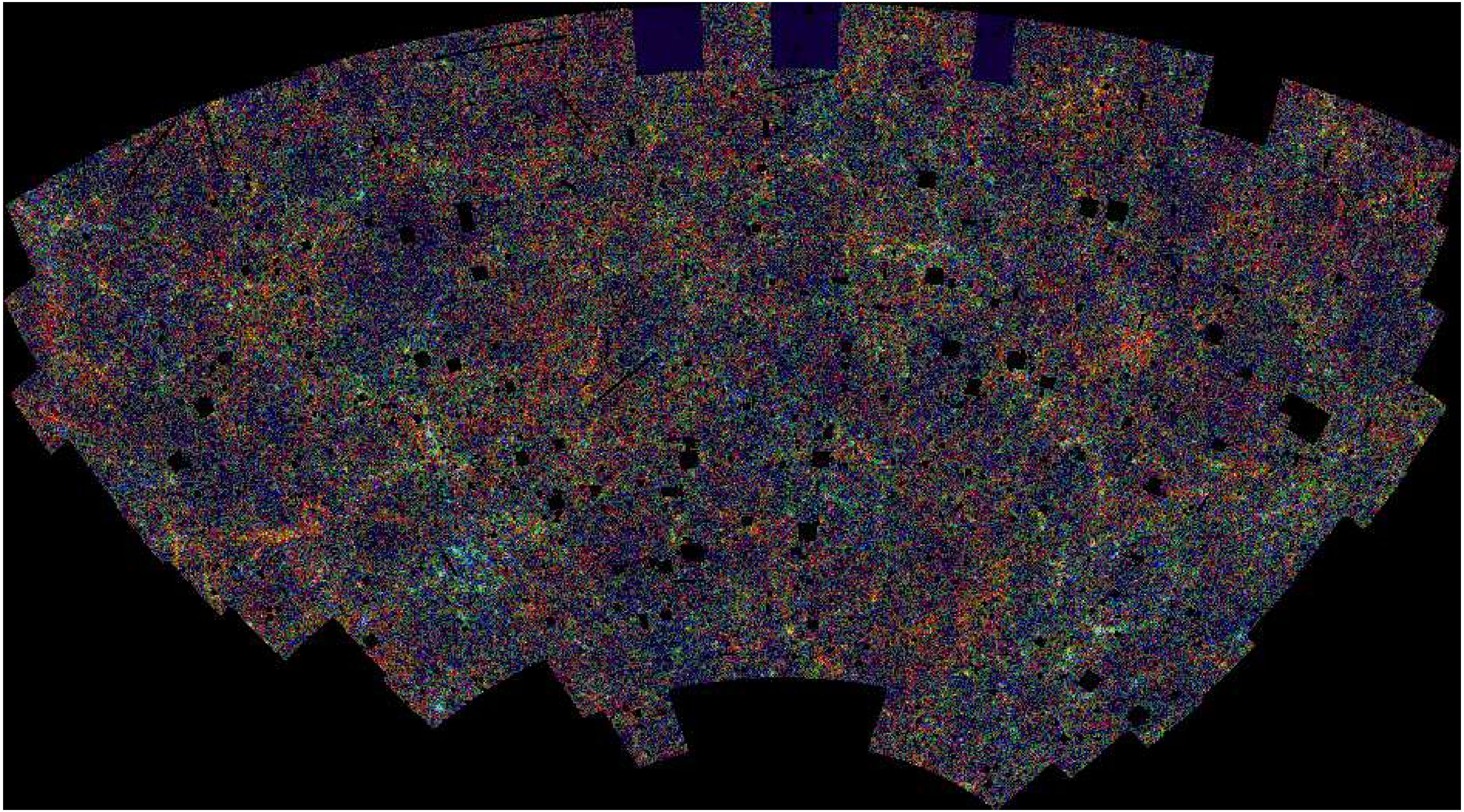
courtesy SDSS



2dF Galaxy Redshift Survey
10h
11h
12h
13h
14h
3h
2h
1h
0h
23h
22h
0.05
0.1
0.15
0.2
redshift
4° slice
63361 galaxies



The complete LCRS survey (at cz large: reach mag. limit)



Galaxies in APM catalogue, color: avg. B in pixel: blue (18) – green (19) – red (20)

Correlation Function, I

Sky surveys show:

Galaxies are *not* evenly distributed: "cosmic web"!

- Structures at scales up to several 10 Mpc
- But: Over-density even in clusters not too dramatic ($\sim$ 100$\times$ denser than average).
- Voids on scales 50 h^{-1} Mpc

$\Longrightarrow$ Need quantitative description of structures.
$\Longrightarrow$ Need physical explanation of structures.
$\Longrightarrow$ Need to understand what we see (do galaxies trace matter distribution??).

Correlation Function, II

Mathematical description of clustering:
Correlation function!

Assume *uniform* distribution of galaxies with galaxy density n (gal Mpc^{-3}).
Chance to find galaxy in volume ΔV:

$$P \propto n\Delta V \tag{8.1}$$

Probability to find galaxies in *two* volumes:

$$P = P_1 \cdot P_2 \propto n^2 \Delta V_1 \Delta V_2 \tag{8.2}$$

Universe inhomogeneous: measure (distance dependent) deviation from mean:

$$P \propto n^2 (1 + \xi(r_{12})) \Delta V_1 \Delta V_2 \tag{8.3}$$

$\xi(r_{12})$ is called the two-point correlation function.

For *small* r:

$$\xi(r) > 0 \Longrightarrow \text{clustering}$$

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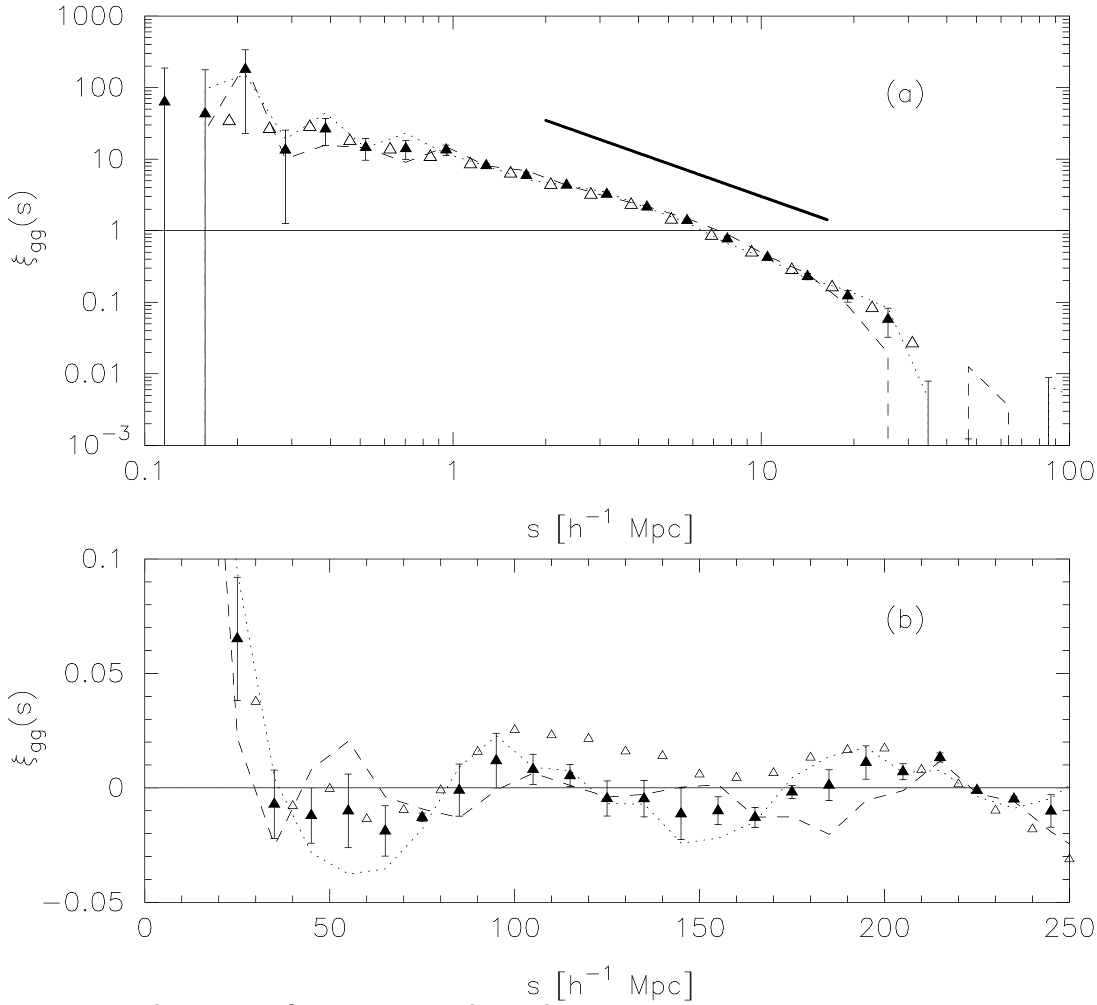
Correlation Function, III



(LCRS; Tucker et al., 1997, Fig. 1)

Rough description: power law

$$\xi(r) = \left(\frac{r}{r_0}\right)^{-\gamma} \tag{8.4}$$

where $r_0 \sim 6\,h^{-1}$ Mpc (correlation length), and $\gamma \sim 1.5 \ldots 1.8$.

Above $r = 30\,h^{-1}$ Mpc: oscillation due to voids.

Correlation Function, IV

ξ is related to the density contrast $\Delta(x)$:
Write density n as

$$n(\mathbf{x}) = n_0(1 + \Delta(\mathbf{x})) \quad \Longleftrightarrow \quad \Delta(\mathbf{x}) = \delta n/n \tag{8.5}$$

Average joint probability to have galaxies at $\mathbf{x}$ and $\mathbf{x}+\mathbf{r}$:

$$P = \langle n(\mathbf{x})\mathrm{d}V_1 \cdot n(\mathbf{x}+\mathbf{r})\mathrm{d}V_2 \rangle \tag{8.6}$$

$$= \left\langle n_0^2(1+\Delta(\mathbf{x}))(1+\Delta(\mathbf{x}+\mathbf{r}))\ \mathrm{d}V_1\mathrm{d}V_2 \right\rangle \tag{8.7}$$

Since $\langle\Delta\rangle = 0$, only cross product survives:

$$= n_0^2\left(1 + \langle\Delta(\mathbf{x})\Delta(\mathbf{x}+\mathbf{r})\rangle\right)\ \mathrm{d}V_1\mathrm{d}V_2 \tag{8.8}$$

where $\langle\ldots\rangle$ denotes averaging over an appropriate volume, i.e.,

$$\langle f(\mathbf{r})\rangle = \frac{1}{V}\int_V f(\mathbf{r})\,\mathrm{d}^3r \tag{8.9}$$

Comparing Eq. (8.8) with Eq. (8.3) shows:

$$\xi(r) = \langle\Delta(\mathbf{x})\Delta(\mathbf{x}+\mathbf{r})\rangle \tag{8.10}$$

$\xi(r)$ is a measure for the average density contrast at places separated by distance r.

Power Spectrum, I

To describe variations: more convenient to work in Fourier space than in "normal" space.

Fourier transform in spatial coordinates defined by:

$$\Delta_r(\mathbf{r}) = \frac{V}{(2\pi)^3} \int \Delta_k \exp(-i\mathbf{k}\cdot\mathbf{r})\, \mathrm{d}^3k \tag{8.11}$$

$$\Delta_k(\mathbf{k}) = \frac{1}{V} \int \Delta(\mathbf{r}) \exp(+i\mathbf{k}\cdot\mathbf{r})\, \mathrm{d}^3r \tag{8.12}$$

But note Parseval's theorem

$$\frac{1}{V}\int \Delta^2(\mathbf{r})\, \mathrm{d}^3x = \frac{V}{(2\pi)^3}\int \Delta_k^2\, \mathrm{d}^3k \tag{8.13}$$

(from signal theory: the power in a time series is the same as the power in the associated Fourier transform)

Left side: variance (mean square amplitude of fluctuations per unit volume)

$\Longrightarrow$ related to power spectrum,

$$P(k) = \Delta_k^2 \tag{8.14}$$

Therefore,

$$\left\langle \Delta^2 \right\rangle = \frac{V}{(2\pi)^3}\int P(k)\, \mathrm{d}^3k \tag{8.15}$$

where (Eq. 8.9)

$$\left\langle \Delta^2 \right\rangle = \frac{1}{V}\int \Delta^2(\mathbf{r})\, \mathrm{d}^3r \tag{8.16}$$

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Power Spectrum, II

How are $\langle\Delta^2\rangle$ and ξ related?

$\Longrightarrow$ Use brute force computation or make use of the correlation theorem.

For functions g, h, the correlation theorem states that the Fourier transform of the correlation,

$$\text{Corr}(g,h) = \int g(x+r)h(r)\,\mathrm{d}x \tag{8.17}$$

is given by

$$\text{FT}\left(\text{Corr}(g,h)\right) = G\,H^* \tag{8.18}$$

where $G = \text{FT}(g)$, etc.

Therefore, setting $g = \Delta(r)$ and $h = \Delta(r)$,

$$\xi(r) = \langle\Delta(\mathbf{x})\Delta(\mathbf{x}+\mathbf{r})\rangle \tag{8.10}$$

$$= \frac{V}{(2\pi)^3}\int |\Delta_k|^2 \exp(i\,\mathbf{k}\cdot\mathbf{r})\,\mathrm{d}^3k \tag{8.19}$$

The power spectrum and ξ are Fourier transform pairs.

(remember Eq. 8.14, $P(k) = \Delta_k^2$!)

See Peebles (1980, sect. 31) for 100s of pages of the properties of ξ, P, etc.

Power Spectrum, III

To better understand ξ and P, assume isotropy for the moment...

We had

$$\xi(\mathbf{r}) \propto \int P(\mathbf{k}) \exp(i\,\mathbf{k}\cdot\mathbf{r})\,\mathrm{d}^3k \tag{8.19}$$

Spherical coordinates in k space:

$$\mathbf{k}\cdot\mathbf{r} = kr\cos\theta \tag{8.20}$$

$$\mathrm{d}V = k^2 \sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi\,\mathrm{d}k \tag{8.21}$$

such that

$$\xi(r) \propto \int_0^\infty \int_0^\pi \int_0^{2\pi} P(k)\exp(ikr\cos\theta)k^2\sin\theta\,\mathrm{d}\phi\,\mathrm{d}\theta\,\mathrm{d}k \tag{8.22}$$

$$= 2\pi \int_0^\infty \int_0^\pi \xi(r)\,\exp(ikr\cos\theta)\,r^2\,\mathrm{d}(\cos\theta)\,\mathrm{d}r \tag{8.23}$$

$$= \frac{V}{2\pi^2}\int_0^\infty P(k)\frac{\sin kr}{kr}\,\mathrm{d}r \tag{8.24}$$

(the last eq. is exact).

For $kr < \pi$: $\sin kr/kr > 0$, while oscillation for $kr > \pi$
$\Longrightarrow$ only wavenumbers $k \lesssim r^{-1}$ contribute to amplitude on scale r.

Since P and ξ are FT pairs, a similar relation holds in the other direction.

Power Spectrum, IV

For a power law spectrum,

$$P(k) \propto k^n \tag{8.25}$$

the correlation function is

$$\begin{aligned} \xi(r) &\propto \int_0^\infty \frac{\sin kr}{kr} k^{n+2}\,\mathrm{d}k \\ &\sim \int_0^{1/r} k^{n+2}\,\mathrm{d}k \\ &\propto r^{-(n+3)} \end{aligned} \tag{8.26}$$

Mass within fluctuation is $M \sim \rho r^3$, i.e., the mass fluctuation spectrum is

$$\xi(M) \propto M^{-(n+3)/3} \tag{8.27}$$

and the rms density fluctuation at mass scale M is

$$\frac{\delta\rho}{\rho} = \xi(M)^{1/2} \propto M^{-(n+3)/6} \tag{8.28}$$

For $n > -3$, the rms mass fluctuations decrease with $M \Longrightarrow$ isotropic universe on largest scales

Power Spectrum, V

What spectra would we expect?

Two simple cases:

Poisson noise: Random statistical fluctuations in number of particles on scale r:

$$\frac{\delta N}{N} = \frac{1}{N} \quad \Longrightarrow \quad \frac{\delta M}{M} = \frac{1}{M} \tag{8.29}$$

and therefore $n = 0$ ($\rho \propto M$!) ("white noise").

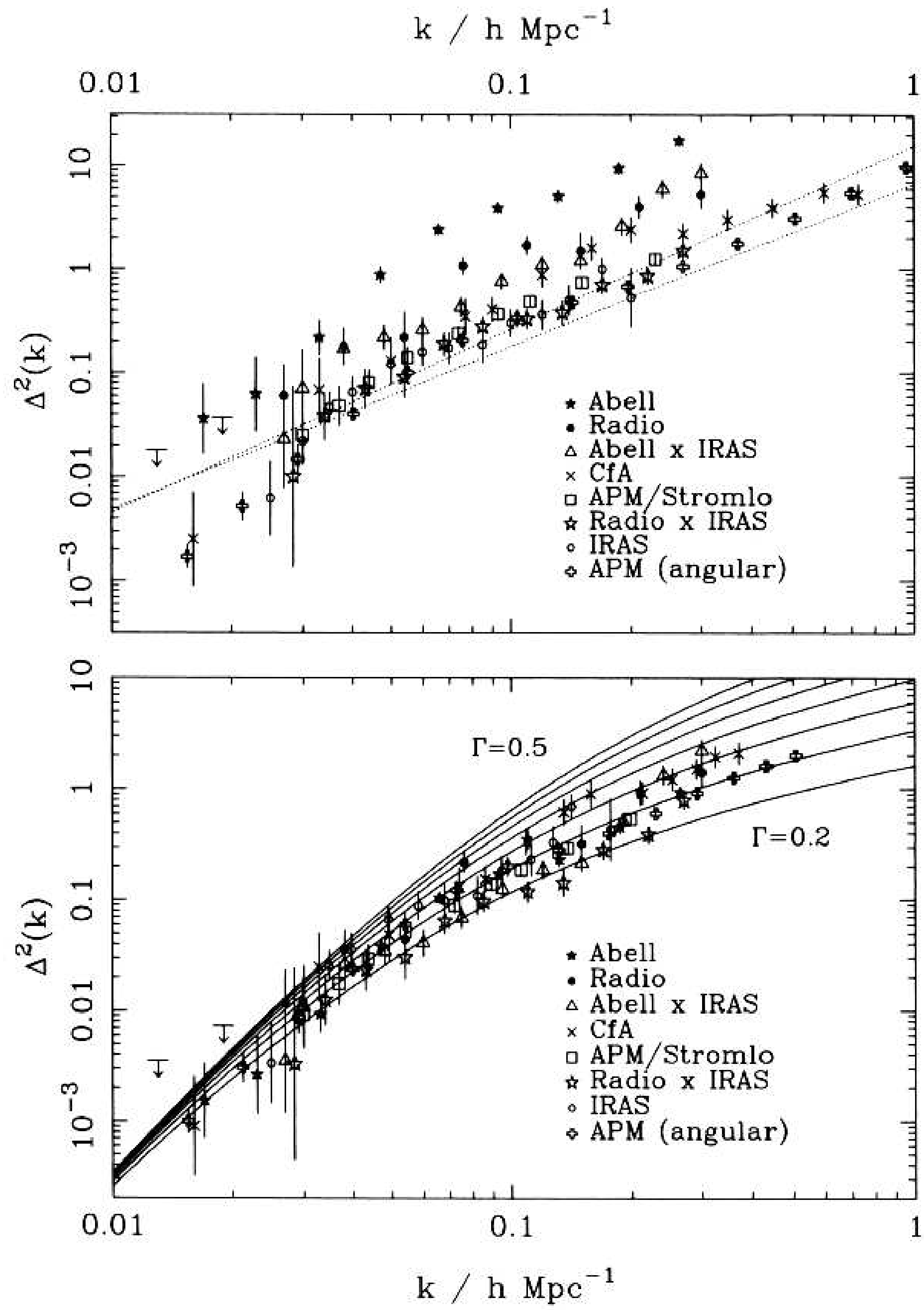
Zeldovich spectrum: defined by $n = 1$. Thus

$$\frac{\delta\rho}{\rho} \propto M^{-2/3} \tag{8.30}$$

...will be important later

The Zeldovich spectrum is the spectrum expected for the case when initial density fluctuations coming through the horizon had the same amplitude.

Power Spectrum, VI



(Peacock, 1999, Fig. 16.4)

Measured power spectrum is more complicated

$\Longrightarrow$ Structure formation to understand details!

Structure formation: Linear Theory, I

Structure formation = evolution of overdensity in universe with time.

Describe density and scale factor wrt normal expansion:

$$\rho(t) = \bar{\rho}(t) \cdot (1 + \delta(t)) \tag{8.31}$$

$$a(t) = \bar{a}(t) \cdot (1 - \epsilon(t)) \tag{8.32}$$

Sign:
$\delta > 0 \Longrightarrow$ *Overdensity*
$\epsilon > 0 \Longrightarrow$ *collapse*

Seek mathematical model for collapse of gravitating material in expanding universe
$\Longrightarrow$ identical to Friedmann equation!
$\Longrightarrow$ Equation describing structure formation:

$$\dot{a}(t) = \frac{8\pi G}{3}\rho(t)a^2(t) + H_0^2(1 - \Omega_0) \tag{8.33}$$

Drop explicit t dependency in the following

Structure formation: Linear Theory, II

Onset of structure formation:

linear regime: $\delta(t)$, $\epsilon(t) \ll 1$

$\Longrightarrow$ Ignore all higher combinations of δ and ϵ.

Left side of Friedmann:

$$\dot{a}^2 = (\dot{\bar{a}} - \dot{\bar{a}}\epsilon - \bar{a}\dot{\epsilon})^2 \tag{8.34}$$

$$= \dot{\bar{a}}^2 - 2\dot{\bar{a}}^2\epsilon - 2\bar{a}\dot{\bar{a}}\,\dot{\epsilon} \tag{8.35}$$

$$= \dot{\bar{a}}^2 - 2\dot{\bar{a}}\frac{\mathrm{d}}{\mathrm{d}t}(\bar{a}\epsilon) \tag{8.36}$$

Right side of Friedmann:

$$\frac{8\pi G}{3}\bar{\rho}(1+\delta)\bar{a}^2(1-\epsilon)^2 + H_0^2(1-\Omega_0) \tag{8.37}$$

$$= \frac{8\pi G}{3}\bar{\rho}\bar{a}^2(1+\delta)(1-2\epsilon) + H_0^2(1-\Omega_0) \tag{8.38}$$

$$= \frac{8\pi G}{3}\bar{\rho}\bar{a}^2(1+\delta-2\epsilon) + H_0^2(1-\Omega_0) \tag{8.39}$$

Now Eq. (8.36)=Eq. (8.39), and subtract terms from Friedmann Equation (eq. 8.33):

$$2\dot{\bar{a}}\cdot\frac{\mathrm{d}}{\mathrm{d}t}(\bar{a}\epsilon) = \frac{8\pi G}{3}\bar{\rho}\bar{a}^2(\delta-2\epsilon) \tag{8.40}$$

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Structure formation: Linear Theory, III

To solve Eq. (8.40): Assume for simplicity $\Omega = 1$, matter-dominated universe.

Matter domination $\Longrightarrow$ $\rho a^3 = \text{const.}$ $\Longrightarrow$

$$\bar{\rho}(1+\delta)\bar{a}^3(1-\epsilon)^3 \sim \bar{\rho}\bar{a}^3(1-3\epsilon+\delta) \stackrel{!}{=} \text{const.} \qquad (8.41)$$

and therefore

$$\epsilon = \delta/3 \qquad (8.42)$$

$\Longrightarrow$ Eq. (8.40) becomes

$$2\dot{\bar{a}} \cdot \frac{\mathrm{d}}{\mathrm{d}t}(\bar{a}\delta) = \frac{8\pi G}{3}\bar{\rho}\bar{a}^2\delta \qquad (8.43)$$

In a $k = 0$ universe,

$$\bar{a}(t) = \left(\frac{3H_0}{2}t\right)^{2/3} =: a_0 t^{2/3} \qquad (4.77)$$

and because of $\rho a^3 = \text{const.}$,

$$\bar{\rho}(t) \propto t^{-2} =: \rho_0 t^{-2} \qquad (8.44)$$

Structure formation: Linear Theory, IV

Insert $\bar{a}$, $\bar{\rho}$ into Eq. (8.43):

$$\frac{4a_0}{3}t^{-1/3}\left(\frac{2a_0}{3}t^{-1/3}\delta + a_0 t^{2/3}\dot{\delta}\right) = \frac{8\pi G}{3}\rho_0 t^{-2} a_0^2 t^{4/3}\delta \tag{8.45}$$

and simplify

$$t^{-2/3}\delta + t^{1/3}\dot{\delta} = 2\pi G\rho_0 t^{-2/3}\delta \tag{8.46}$$

$$t\dot{\delta} + (1 - 2\pi G\rho_0)\delta = 0 \tag{8.47}$$

The general solution of Eq. (8.47) is a power-law
$\Longrightarrow$ Growth of structure!

Since also *negative* PL indexes possible $\Longrightarrow$ Some initial perturbations are damped out!

Need better theory to do that in detail. . .

Structure formation: Linear Theory, V

Better linear theory: Use linearized equations of motion from hydrodynamics to compute gravitational collapse

Detailed theory very difficult

see handout for a few ideas of what is going on...

Classical approach:

Consider sphere of material:

Potential energy of sphere:

$$U = -\frac{1}{2}\int \rho(x)\Phi(x)\,\mathrm{d}^3x \sim -\frac{16\pi^2}{15}G\rho^2 r^5 \tag{8.48}$$

Total kinetic energy content:

$$T \sim \frac{c_\mathrm{s}^2}{2}\frac{4\pi r^3\rho}{3} \tag{8.49}$$

c_s: speed of sound; for neutral Hydrogen, $c_\mathrm{s} = \sqrt{5T/3m_\mathrm{p}}$.

Sphere collapses if $|U| > T$, i.e., when

$$2r \gtrsim \sqrt{\frac{5}{2\pi}}\sqrt{\frac{c_\mathrm{s}^2}{G\rho}} \sim c_\mathrm{s}\sqrt{\frac{\pi}{G\rho_0}} =: \lambda_\mathrm{J} \tag{8.50}$$

λ_J is called the Jeans length, the corresponding mass is the Jeans mass,

$$M_\mathrm{J} = \frac{\pi}{6}\rho\lambda_\mathrm{J}^3 \tag{8.51}$$

Structures with $m < M_\mathrm{J}$ cannot grow.

Note that c_s is time dependent $\Longrightarrow$ M_J can change with time!

A better derivation of the Jeans length comes from considering the evolution of a fluid in an expanding universe. Assuming that the initial density perturbations were small, we can use perturbation theory for obtaining deviations from homogeneity (=structures).

In a Friedmann universe, for length scales $< 1/H$, dynamical equations are Newtonian to first order, but we need to still use the scale factor, $a(t)$ in the fluid equations.

Continuity equation:

$$\dot{\rho} + \nabla \cdot (\rho \mathbf{v}) = 0 \tag{8.52}$$

Euler's equation:

$$\dot{\mathbf{v}} + (v \cdot \nabla) v = -\nabla \left(\Phi + \frac{\rho}{c} \right) \tag{8.53}$$

Poisson's equation:

$$\nabla^2 \Phi = 4\pi G \rho \tag{8.54}$$

Without perturbations (i.e., the zeroth order solution) is given by the normal Friedmann solutions:

$$\rho_0(t, \mathbf{r}) = \frac{\rho_0}{a^3(t)} \quad \text{(dilution by expansion)} \tag{8.55}$$

$$\mathbf{v}_0(t, \mathbf{r}) = \frac{\dot{a}(t)}{a(t)} \mathbf{r} \quad \text{(Hubble law)} \tag{8.56}$$

$$\Phi_0(t, \mathbf{r}) = \frac{2\pi G \rho_0 r^2}{3} \quad \text{(soln. of Poisson with } \rho = \text{const.)} \tag{8.57}$$

Convert into comoving coordinates ($\mathbf{x} = \mathbf{r}/a(t)$) to get rid of the $a(t)$'s and write down perturbation equations:

$$\rho(t, \mathbf{x}) = \rho_0(t) + \rho_1(t) =: \rho_0(t)\,(1 + \delta(t, \mathbf{x})) \tag{8.58}$$

$$\mathbf{v}(t, \mathbf{x}) = \mathbf{v}_0(t, \mathbf{x}) + \mathbf{v}_1(t, \mathbf{x}) \tag{8.59}$$

$$\Phi(t, \mathbf{x}) = \Phi_0(t, \mathbf{x}) + \Phi_1(t, \mathbf{x}) \tag{8.60}$$

where $|\delta|$, $|\mathbf{v}_1|$, $|\Phi_1|$ small (δ is called density perturbation field).

Since the equations are spatially homogeneous, we can Fourier transform them to search for plane wave solutions. The general perturbation solution can then later be found by performing linear combinations of these plane waves.

$$\delta(t, \mathbf{x}) = \frac{1}{(2\pi)^3} \int e^{i\mathbf{k}\cdot\mathbf{x}} \delta(t, \mathbf{k}) d^3 k \Longleftrightarrow \delta(t, \mathbf{k}) = \int e^{-i\mathbf{k}\cdot\mathbf{x}} \delta(t, \mathbf{x}) d^3 x \tag{8.61}$$

Inserting into hydro equations gives

$$\ddot{\delta}(t, \mathbf{k}) + 2\frac{\dot{a}(t)}{a(t)} \dot{\delta}(t, \mathbf{k}) + \left(\frac{k^2 c_s^2}{a^2(t)} - 4\pi G \rho_0 \right) \delta(t, \mathbf{k}) = 0 \tag{8.62}$$

where the sound speed is $c_s^2 = (\partial p / \partial \rho)_{\text{adiabatic}}$.

Solutions to eq. 8.62 grow or decrease depending on sign of

$$\kappa_J = \left(\frac{k^2 c_s^2}{a^2(t)} - 4\pi G \rho_0 \right) \tag{8.63}$$

Thus, growth is only possible for $k > k_\mathrm{J}$ where

$$k_\mathrm{J} = \sqrt{\frac{4\pi G\rho_0 a^2(t)}{c_\mathrm{s}^2}} \tag{8.64}$$

or, in terms of physical wavelengths,

$$\lambda_\mathrm{J} = \frac{2\pi a(t)}{k_\mathrm{J}} = c_\mathrm{s}\sqrt{\frac{\pi}{G\rho_0}} \tag{8.65}$$

the Jeans length.

Stages of Structure Formation

Early universe: radiation dominates:

$$c_\mathrm{s} = c/\sqrt{3} \quad \text{and} \quad \rho_\mathrm{r} c^2 = \sigma T^4 \tag{8.66}$$

and therefore

$$\lambda_\mathrm{J,rad} = c^2 \sqrt{\pi 3 G \sigma T^4} \propto a^2 \quad \text{and} \quad M_\mathrm{J} \propto \rho_\mathrm{m} \lambda^3_\mathrm{J,rad} \propto a^3 \tag{8.67}$$

In the early universe, the Jeans mass grows quickly.

At time of radiation – matter equilibrium,

$$\rho_\mathrm{m} = \rho_\mathrm{rad} = \sigma T^4_\mathrm{eq}/c^2 \tag{8.68}$$

and

$$M_\mathrm{J}(t_\mathrm{eq}) = \frac{\pi^{5/2}}{18\sqrt{3}} \frac{c^4}{G^{3/2}\sigma^{1/2}} \frac{1}{T_\mathrm{eq}} \sim \frac{3.6 \times 10^{16} (\Omega_0 h^2)^{-2}\, M_\odot}{(T/T_\mathrm{eq})^3} \tag{8.69}$$

assuming $1 + z_\mathrm{eq} = 24000\, \Omega_0 h^2$.

$\Longrightarrow$ much larger than mass in galaxy cluster (about mass in cube with 50 Mpc side length $\Longrightarrow$ size of voids!)

Overdense regions with $m < M_\mathrm{J,rad}$ are smoothed out by the radiation coupling to matter.

Much larger structures also cannot grow since λ is larger than horizon radius $\Longrightarrow$ Mass spectrum of possible structures.

Stages of Structure Formation

After t_{eq} not much happens until $T_{\mathrm{rec}} \sim 3000\,\mathrm{K}$

$\Longrightarrow$ recombination

$\Longrightarrow$ Sound speed drops dramatically (radiation and matter decouple):

$$c_{\mathrm{s}} \sim \frac{kT}{m_{\mathrm{p}}} \sim 5\,\mathrm{km\,s^{-1}} \tag{8.70}$$

$\Longrightarrow$ M_{J} drops by 10^{11}:

$$M_{\mathrm{J,eq}} = \frac{\pi\bar{\rho}}{6}\left(\frac{\pi k T_{\mathrm{rec}}}{G\bar{\rho}m_{\mathrm{p}}}\right)^{1/2} \sim 5\times 10^{5}(\Omega_0 h^2)^{-1/2}\,M_\odot \tag{8.71}$$

after that, M_{J} drops because of expansion.

So, in pure matter universe:

- at begin: huge structures form (Zeldovich pancakes)
- suddenly at recombination: fragmentation

$\Longrightarrow$ top-down model

Problem: Not really what has been observed

Solution: Dark matter

Stages of Structure Formation

Structure formation with dark matter:

DM unaffected by radiation pressure $\Longrightarrow$ collapse of smaller structures possible $\Longrightarrow$ bottom-up model

As long as DM relativistic:

$$M_{\mathrm{J,HDM}} = \frac{\pi \rho_{\mathrm{DM}}}{6} \left(\frac{\pi c_{\mathrm{DM}}}{G \rho_{\mathrm{DM}}} \right)^{3/2} \tag{8.72}$$

Hot Dark Matter: $c_{\mathrm{HDM}} \sim c/\sqrt{3}$

Cold Dark Matter: $c_{\mathrm{CDM}} \ll c/\sqrt{3}$

Standard CDM Scenario:

- DM cools long before t_{rec}
- CDM structures form, M_{J} about galaxy mass, while baryons coupled to radiation $\Longrightarrow$ stays smooth
- t_{rec}: matter decouples, falls in DM gravity wells

CDM "seeds" structures!

Gives not exactly observed power spectrum $\Longrightarrow$ Currently preferred: combination of CDM and ΛDM

UWarwick

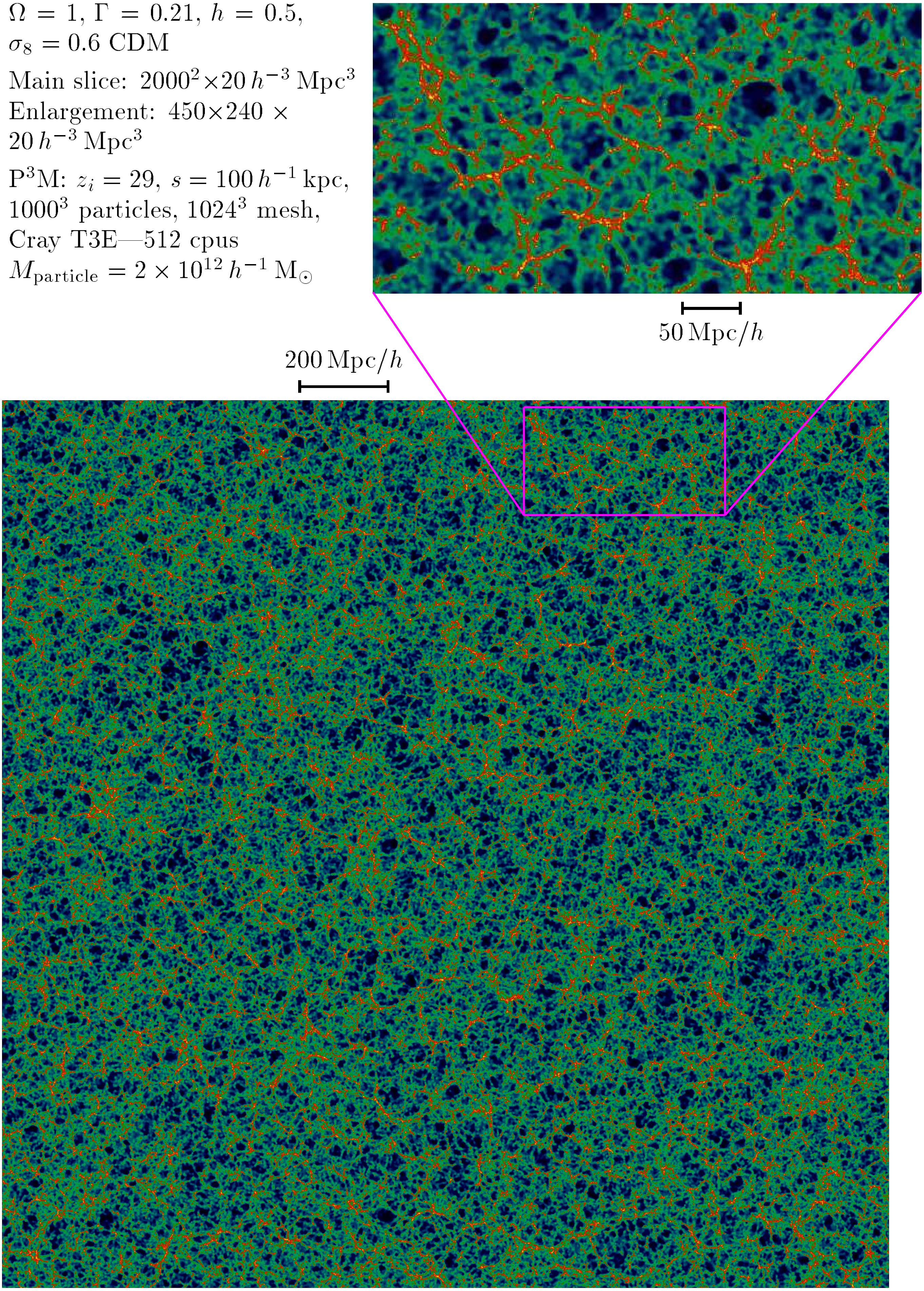
Stages of Structure Formation

Finally, the *real* linear theory has to be done in linearized or even full general relativity
$\Longrightarrow$ very, very complicated.

Full fledged, detailed structure formation is mainly done numerically.

N-body codes: describe particles (=galaxies) as points, compute mutual interactions in expanding universe
Requires massive computing power.
VIRGO consortium: U.S.A., Canada, Germany, UK
Hubble Volume Simulation: Garching T3E (512 processors), 70 h CPU time
Show some results on following slides and movies.

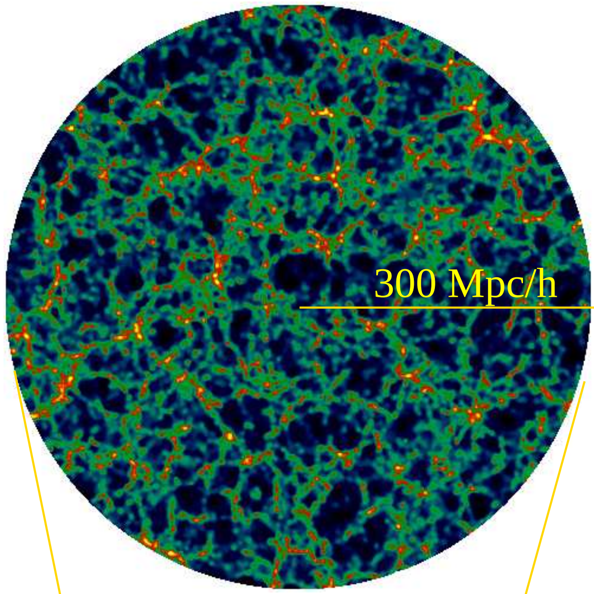
`http://www.mpa-garching.mpg.de/~virgo/virgo/`

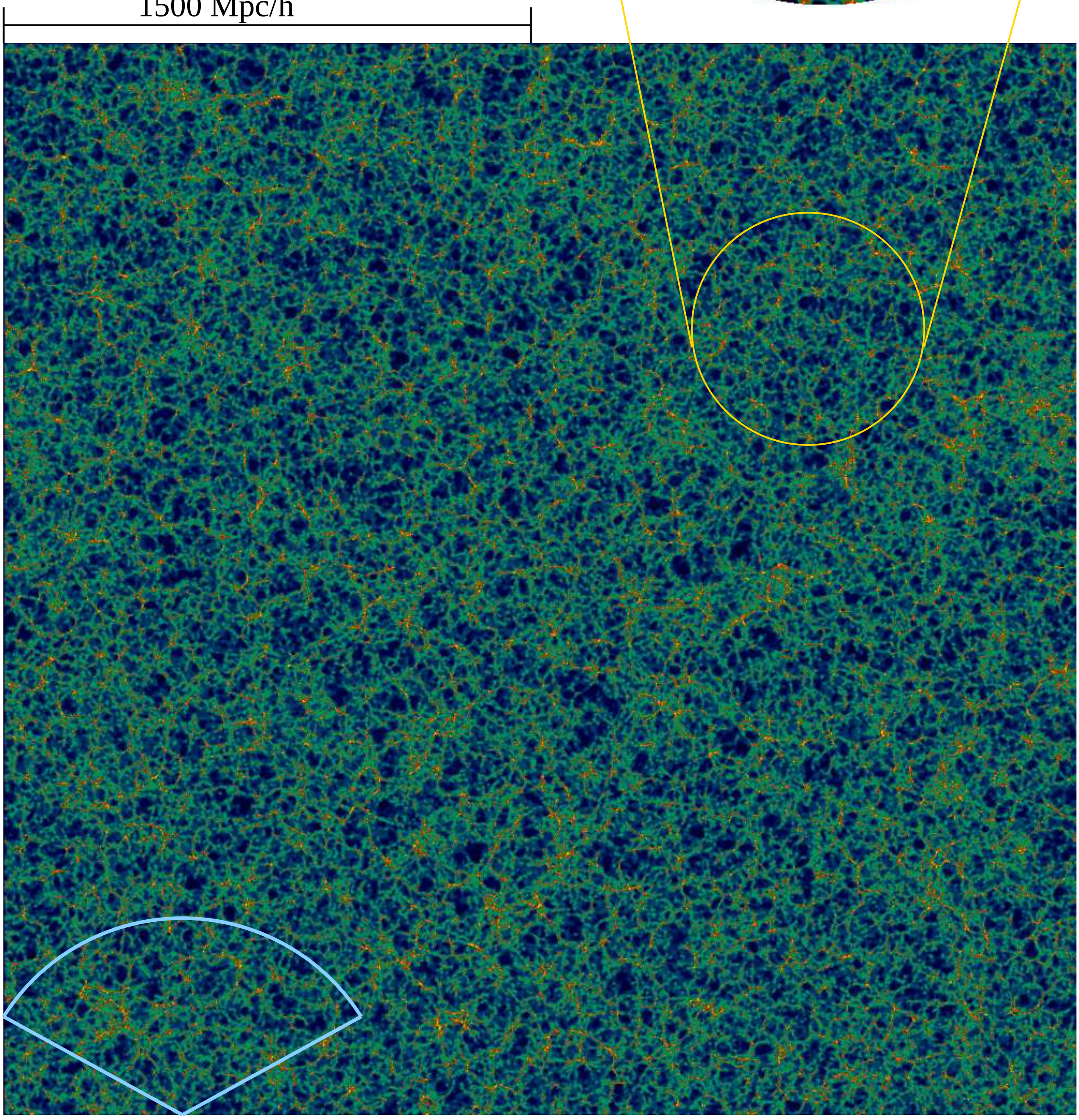


Standard CDM

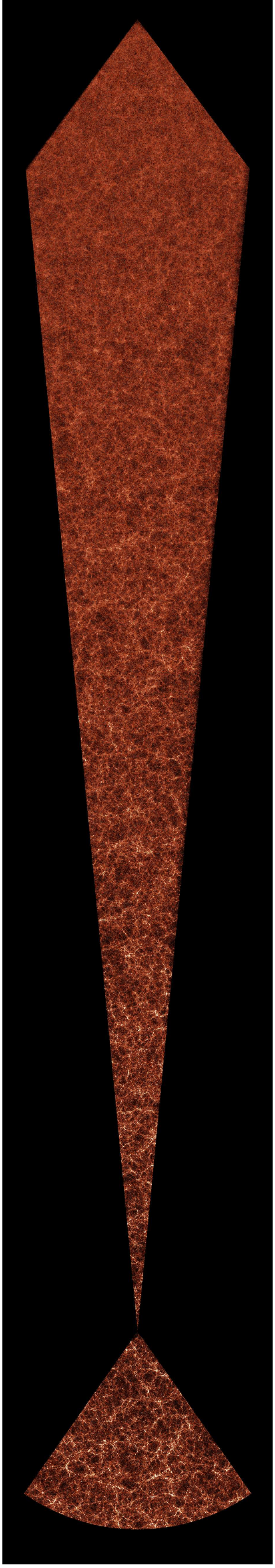
The Hubble Volume Simulation

Ω=0.3,Λ=0.7,h =0.7,
σ_8=0.9 (ΛCDM)
3000 x 3000 x 30 h^{-3}Mpc^3
P^{3}M: z_i=35, s=100 h^{-1}kpc
1000^3 particles, 1024^3 mesh
T3E(Garching) - 512cpus
$M_{\mathbf{particle}}$ = 2.2 x $10^{12} h^{-1} M_{\mathbf{sol}}$

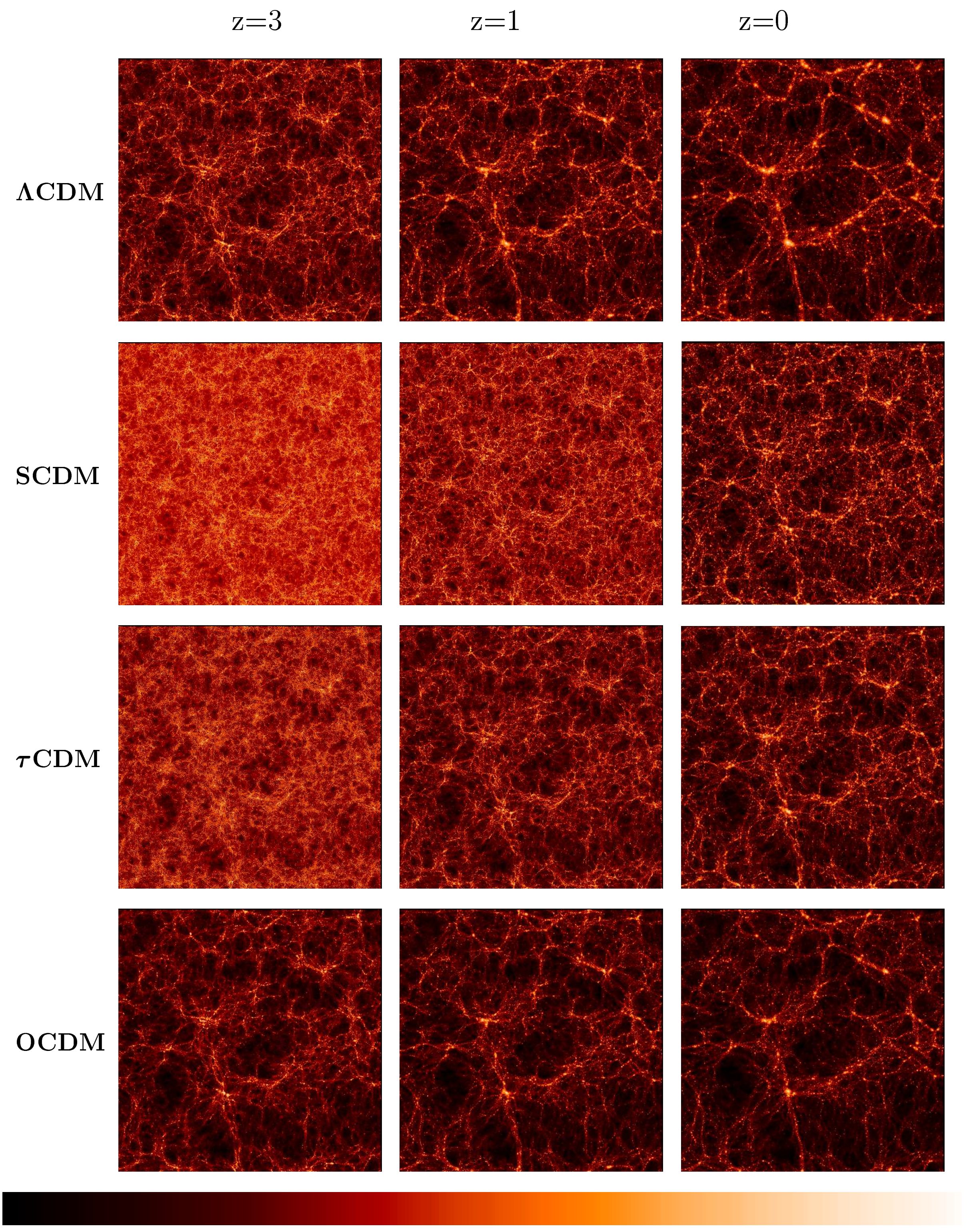




ΛDM

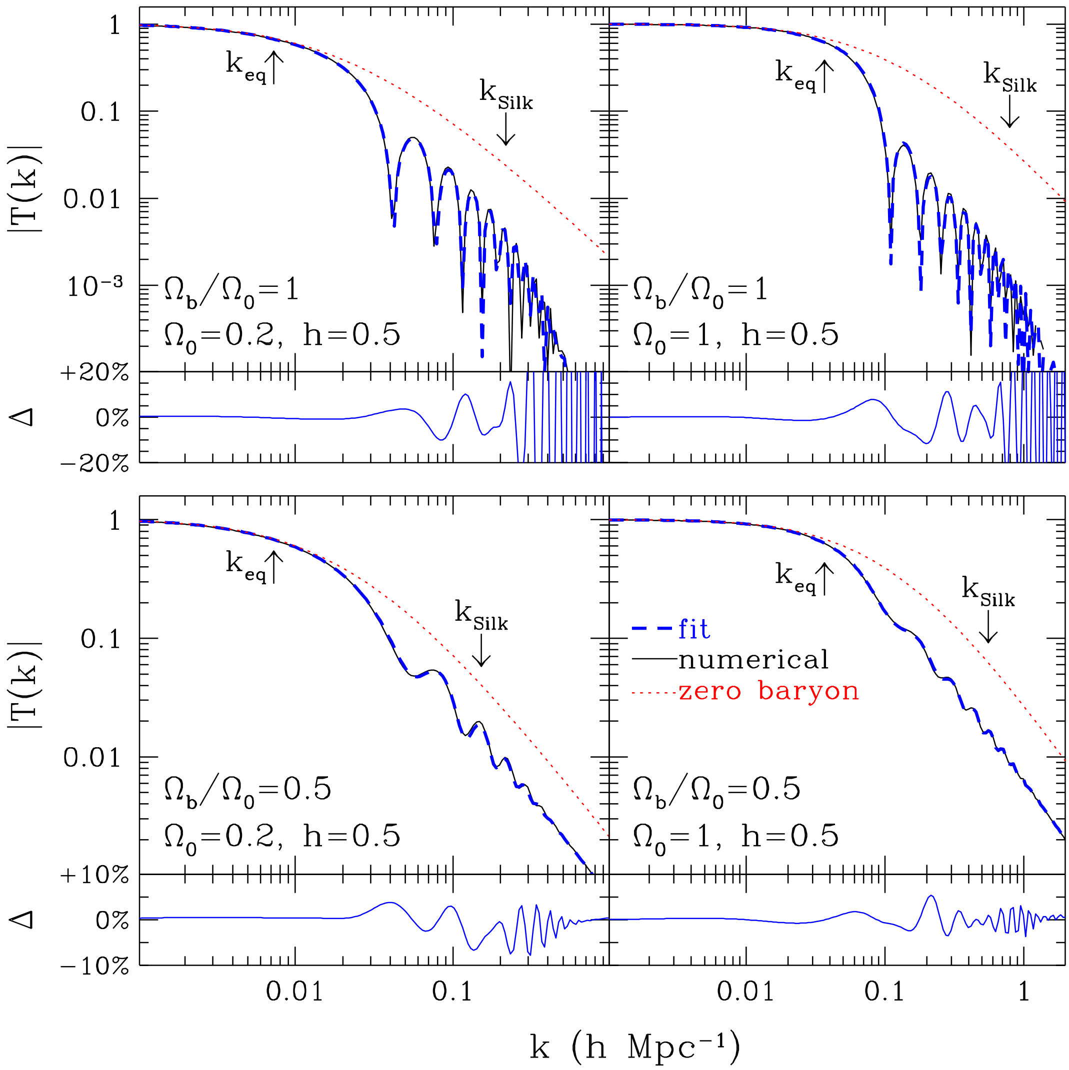


Evolution of clustering along light cone



The VIRGO Collaboration 1996

Formal Structure Formation



Eisenstein & Hu, 1997

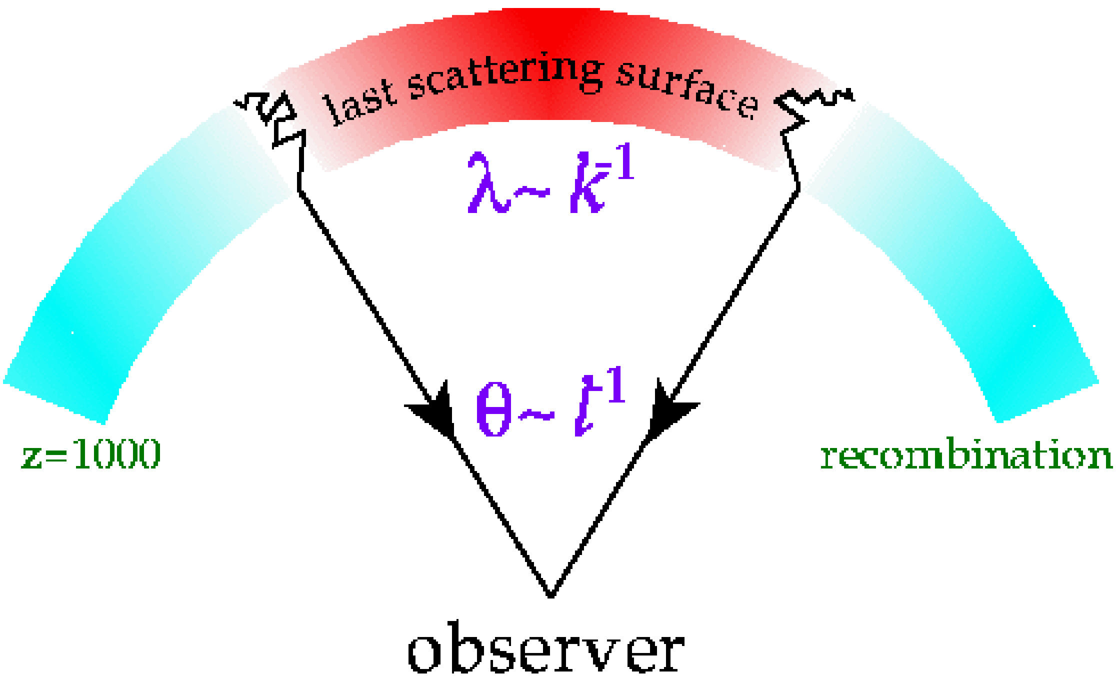
Computation of real power spectra difficult: growth under self-gravitation pressure effects dissipation.

To predict observations from today: define transfer function

$$\delta_k(z=0) = \delta_k(z)D(z)T_k \qquad (8.73)$$

But: need initial conditions, $\delta_k(z)$!

CMBR



courtesy Wayne Hu

Matter and Radiation are coupled, i.e., large mass density $=$ high photon density.

Photons from overdense regions: gravitational redshift $\Longrightarrow$ observable!

(Sachs Wolfe Effect)

CMBR: Radiation from surface of last scattering

CMBR Fluctuations trace gravitational potential at $z \sim 1100$!

CMBR

Temperature fluctuations:

$$\frac{\Delta T}{T} \sim \frac{\Delta \Phi_{\mathrm{g}}}{c^2} \tag{8.74}$$

where

$$\Delta \Phi_{\mathrm{g}} \sim -\frac{2G\Delta M}{R} = \frac{8\pi G}{3}\bar{\rho} R^2 \delta \tag{8.75}$$

$$= -\delta(t)\left(H(t)R\right)^2 \tag{8.76}$$

Current angle of region on sky:

$$\alpha \sim R/d_{\mathrm{A}} \tag{8.77}$$

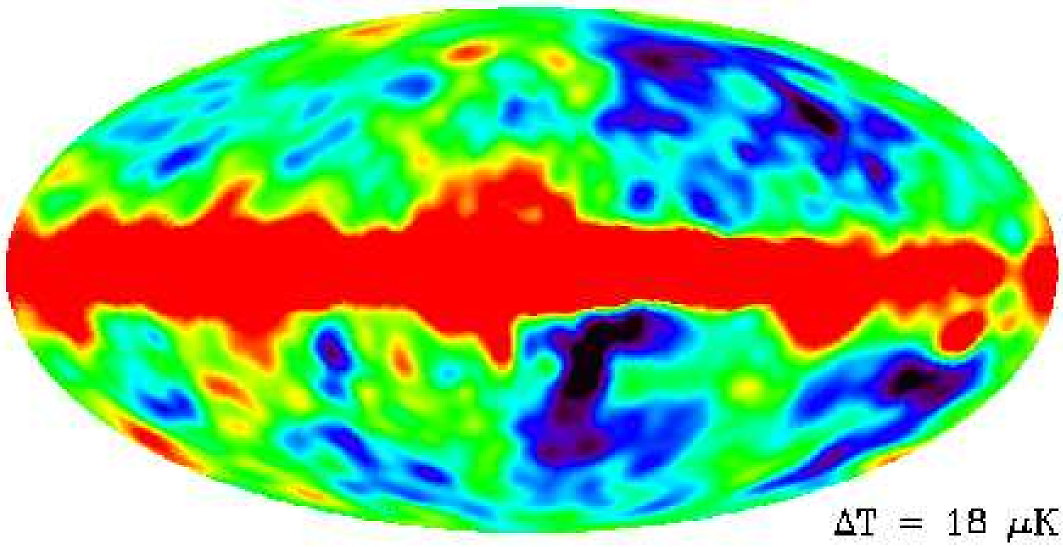
where the angular diameter distance

$$d_{\mathrm{A}} = d_{\mathrm{L}}/(1+z)^2 \tag{8.78}$$

Therefore:

$$\frac{\Delta T}{T} \sim \frac{\Delta \Phi_{\mathrm{g}}}{c^2} \propto -\delta \alpha^2 \tag{8.79}$$

CMBR



Eqs. (8.76) and (8.79) imply

$$\frac{\Delta T}{T} \sim -\frac{\delta\alpha^2}{3} \qquad (8.80)$$

Quotient 3 from more detailed theory, "Integrated Sachs Wolfe effect"

COBE: Resolution $\alpha \sim 7^\circ$ (corresponds to $\sim 10^{20}\,M_\odot$ at recombination)

COBE results imply $\delta \sim 10^{-3}$ at recombination

This is small for pure matter dominated universe

$\Longrightarrow$ Implies existence of dark matter!

UWarwick

CMBR

Expand CMB fluctuations on sky in spherical harmonics:

$$\frac{\Delta T}{T}(\theta,\phi)=\sum_{\ell,m} a_{\ell,m} Y_{\ell,m}(\theta,\phi) \tag{8.81}$$

Since rotationally symmetric, can express variation in terms of multipole coefficients, C_ℓ:

$$C(\theta)=\frac{1}{4\pi}\sum_{\ell}\sum_{m=-\ell}^{+\ell}|a_{\ell,m}|P_\ell(\cos\theta) \tag{8.82}$$

$$=: \frac{1}{4\pi}\sum_{\ell}(2\ell+1)C_\ell P_\ell(\cos\theta) \tag{8.83}$$

where $C(\theta)=\langle\Delta T/T\rangle$ and where the P_ℓ are the Legendre polynomials.

CMBR

Expect following features:

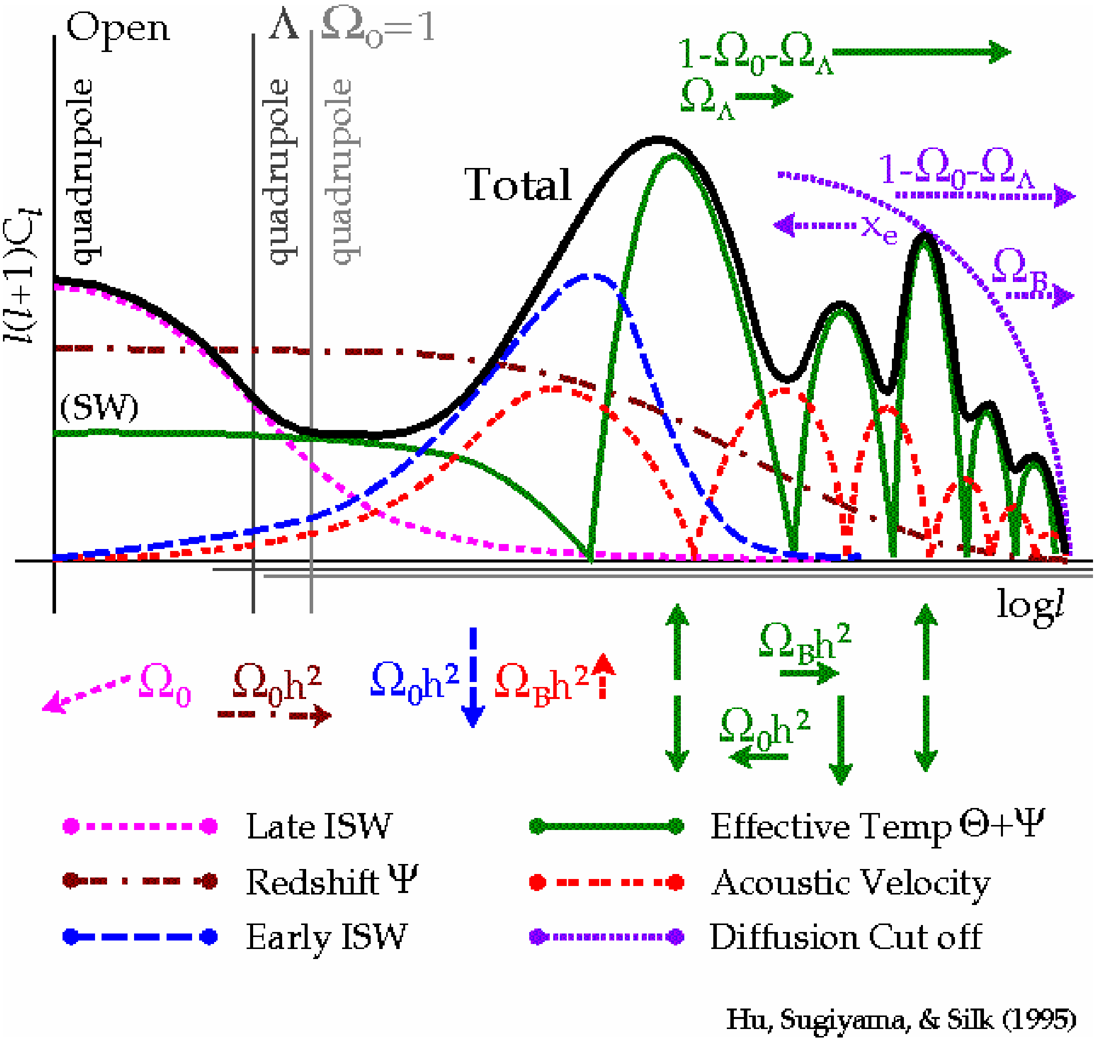
Large angle anisotropy: (small ℓ, scales $\gtrsim$ horizon at decoupling): Flat part due to Sachs-Wolfe effect

Smaller angular scales: (larger ℓ): Influenced by photon-baryon interactions: Matter falls in potential well
$\Longrightarrow$ Pressure resists
$\Longrightarrow$ acoustic oscillations
$\Longrightarrow$ Power at selected scales!

Power from those density fluctuations which had their maximum amplitude at time of last scattering dominates $\Longrightarrow$ acoustic peak

Also damping from photon diffusion (Compton scattering; Silk damping [after Joseph Silk])

CMBR



Location and strength of acoustic peaks dependent on

$$\Omega_b \quad H_0 \quad \Omega_0$$

Position of acoustic peak not observed with COBE (at smaller scale than 7°)

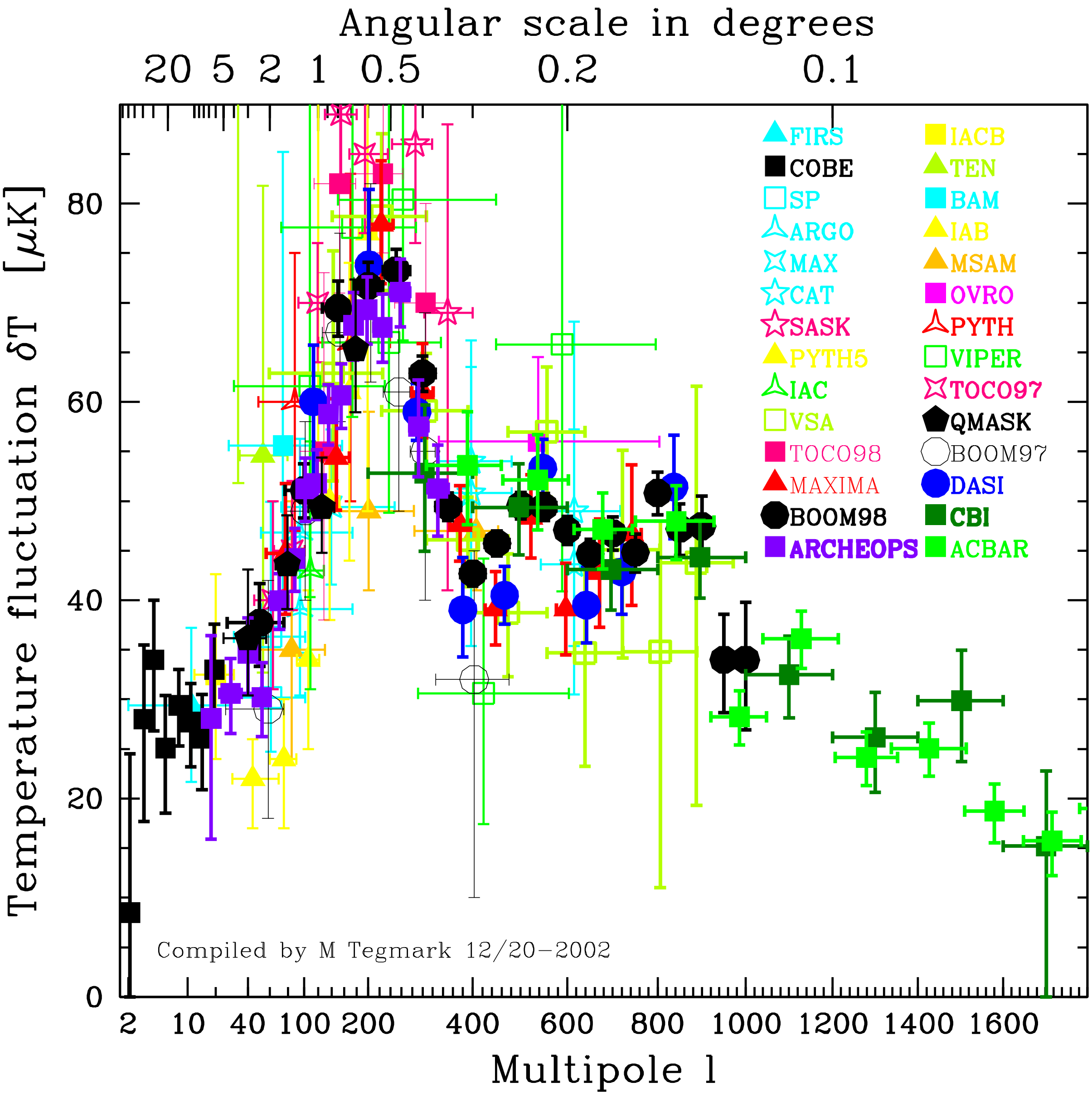
courtesy BOOMERANG team

Enter: BOOMERANG (Balloon Observations of Milimetric Extragalactic Radiation and Geophysics), Flight in Antarctica 1998 December 29 – 1999 January 9

BOOMERANG before Mt. Erebus; courtesy BOOMERANG team

Other balloon missions: MAXIMA-1,. . .

Summary: Pre-WMAP



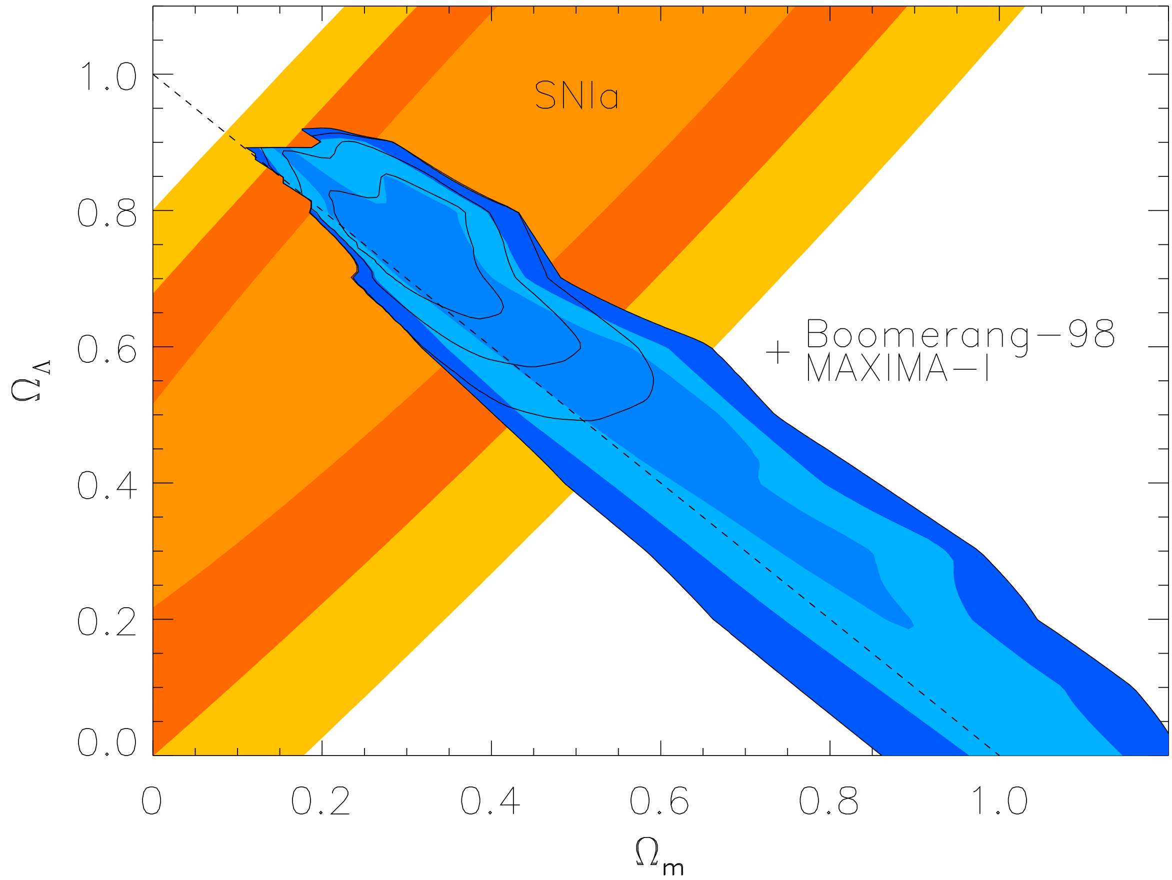
Courtesy M. Tegmark

1st acoustic peak found by BOOMERANG in 1999

(Jaffe et al., 2000)

. . . confirmed by many experiments since then

Summary: Pre-WMAP



(Jaffe et al., 2000, black contours: incl. Large Scale Structure)

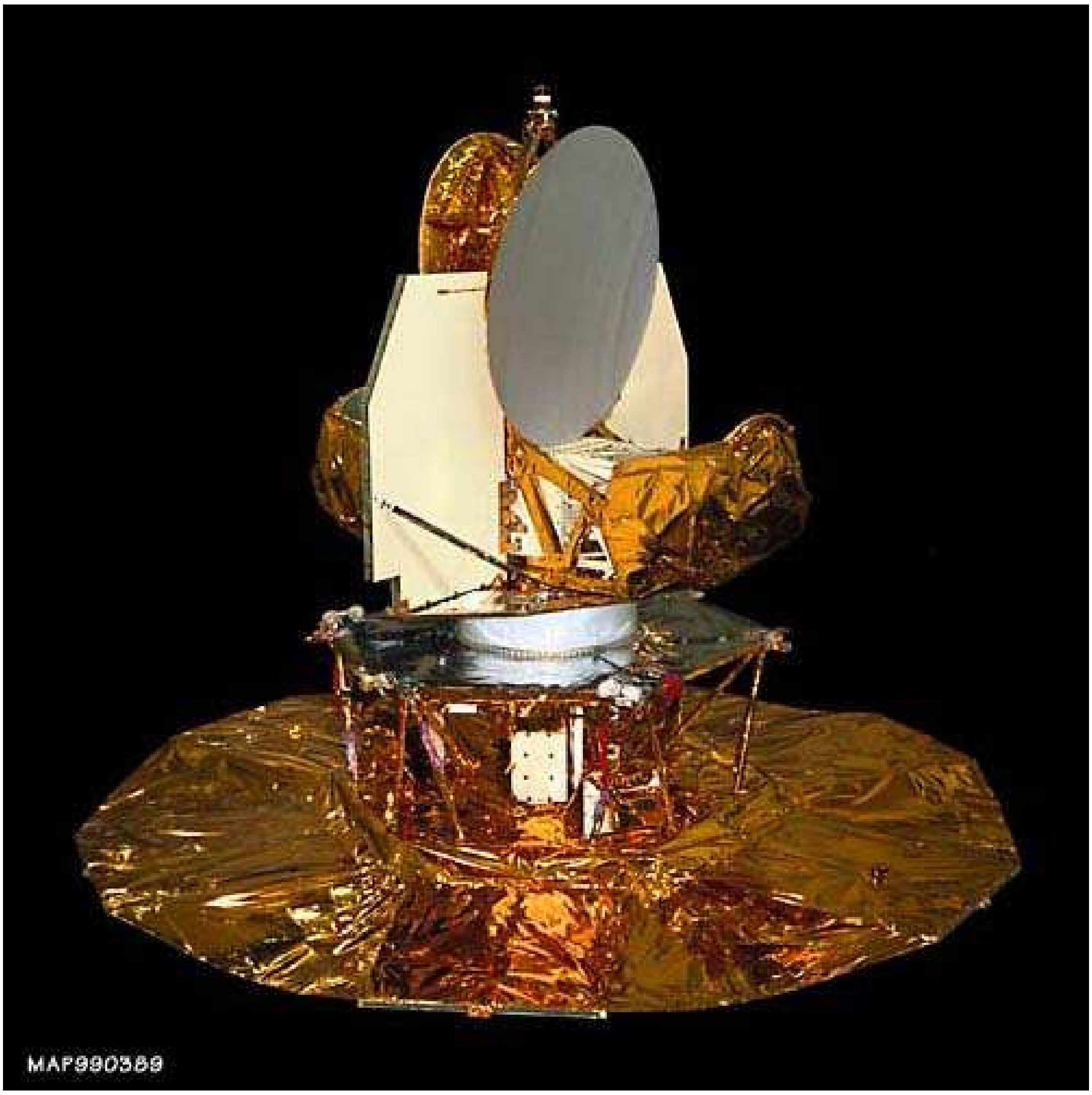
General summary of CMB fluctuations (COBE, BOOMERANG, MAXIMA):

$$\Omega_{\rm tot} \simeq 1.11 \pm 0.07 \left({}^{+0.13}_{-0.12}\right) \tag{8.84}$$

and

$$\Omega_b h^2 \simeq 0.032^{+0.005}_{-0.004} \left({}^{+0.009}_{-0.008}\right) \tag{8.85}$$

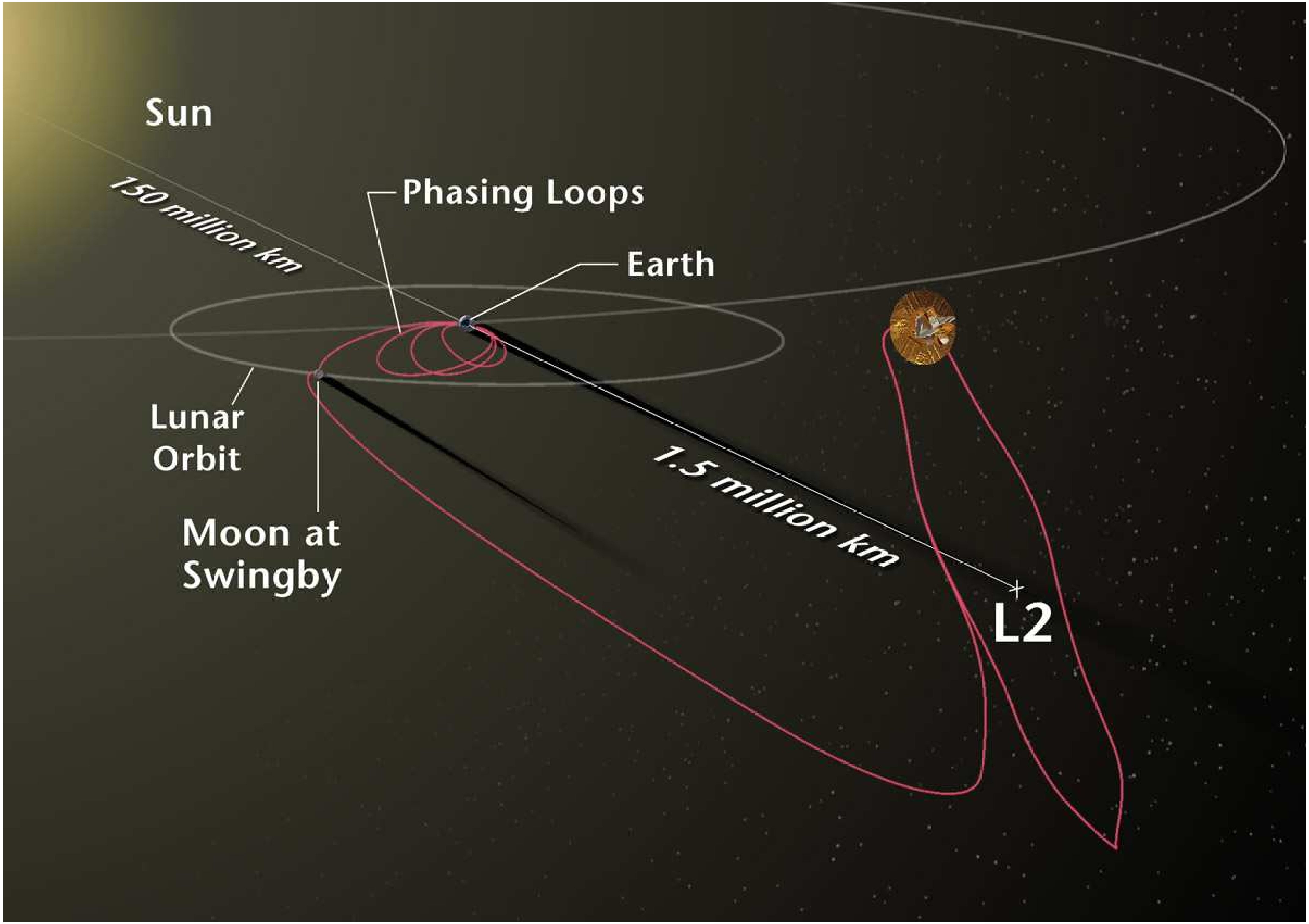
WMAP



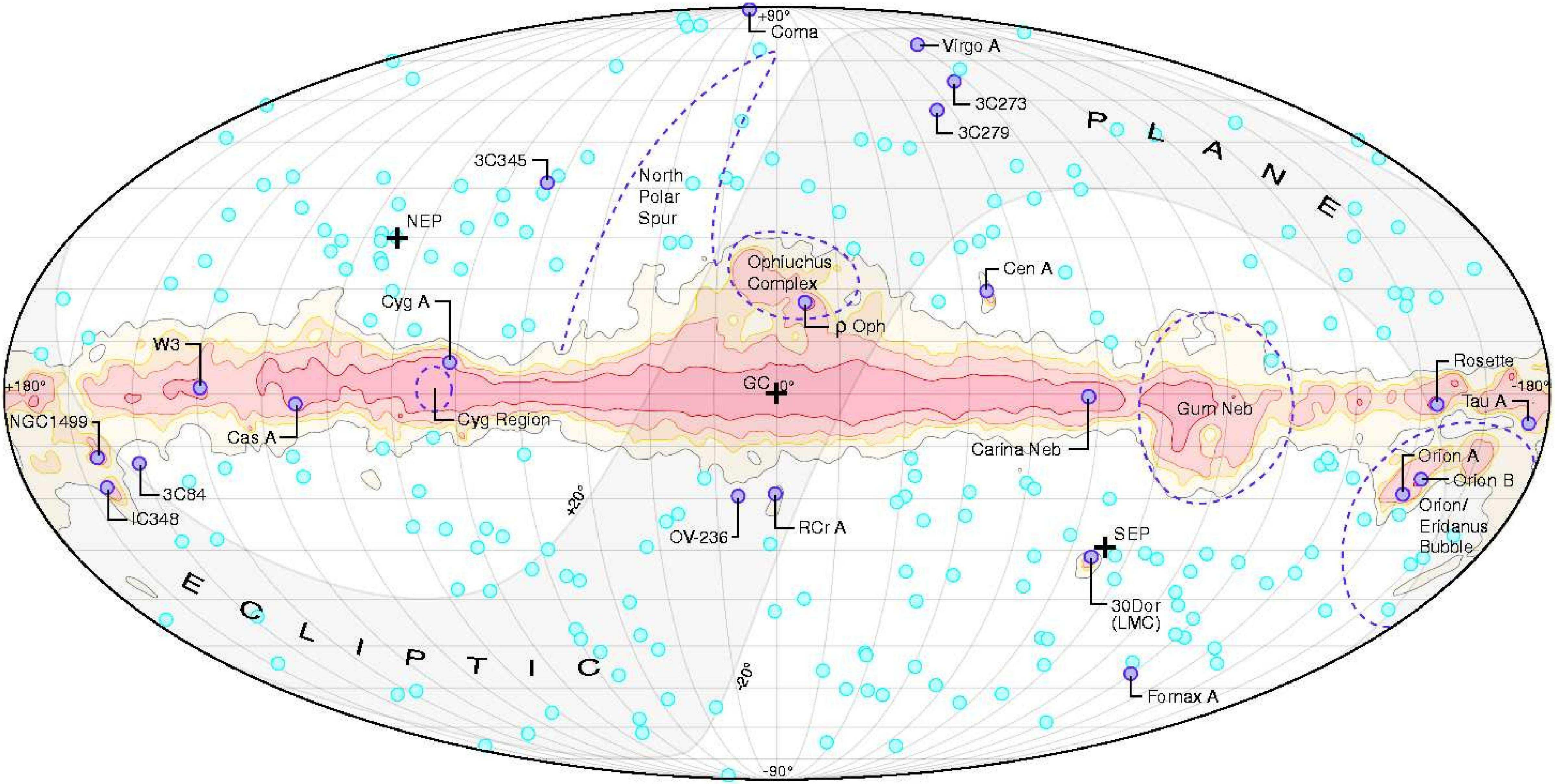
Wilkinson Microwave Anisotropy Probe (WMAP):

- Launched 2001 June 30, measurements began 2001 August 10
- Orbit around 2nd Lagrange Point of Sun-Earth System
- Highly precise radiometers of high spatial resolution (best: 0.21° FWHM in W-Band at 3.2 mm) in five wavebands

(see Bennett et al. 2003 for an overview).

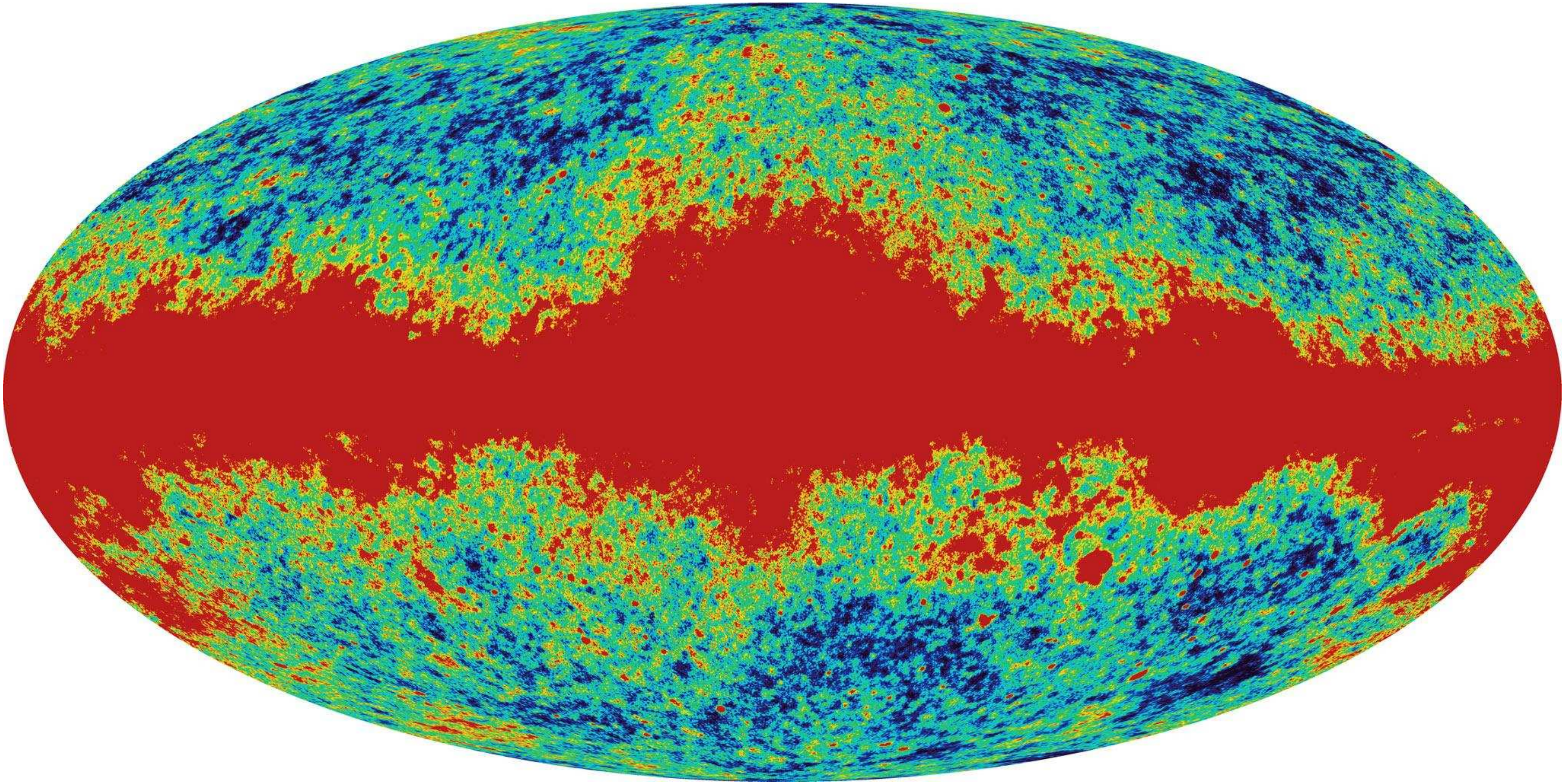


Sun
150 million km
Phasing Loops
Earth
Lunar Orbit
Moon at Swingby
1.5 million km
L2

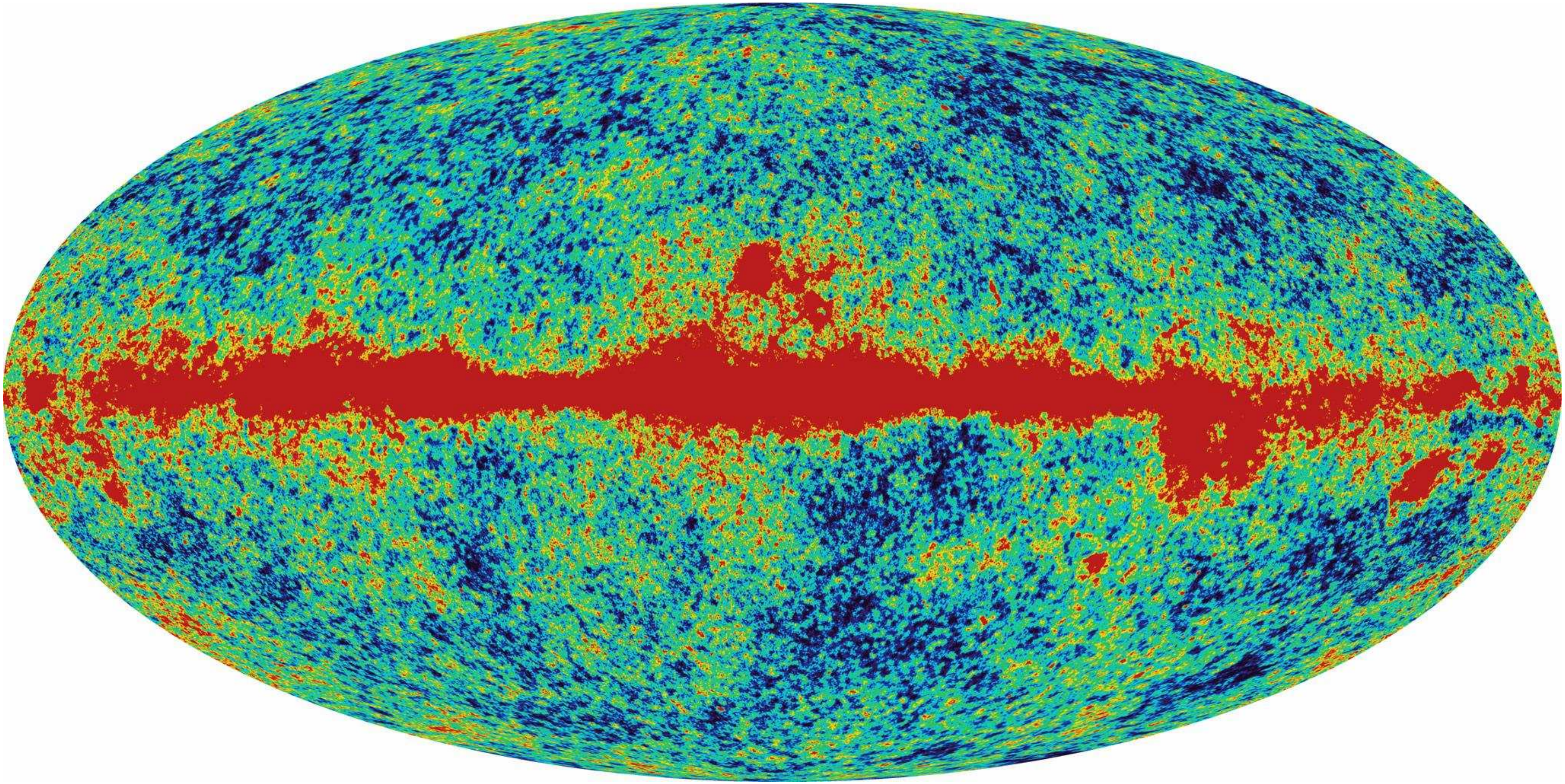


Foreground features of the microwave sky (Bennett et al., 2003).

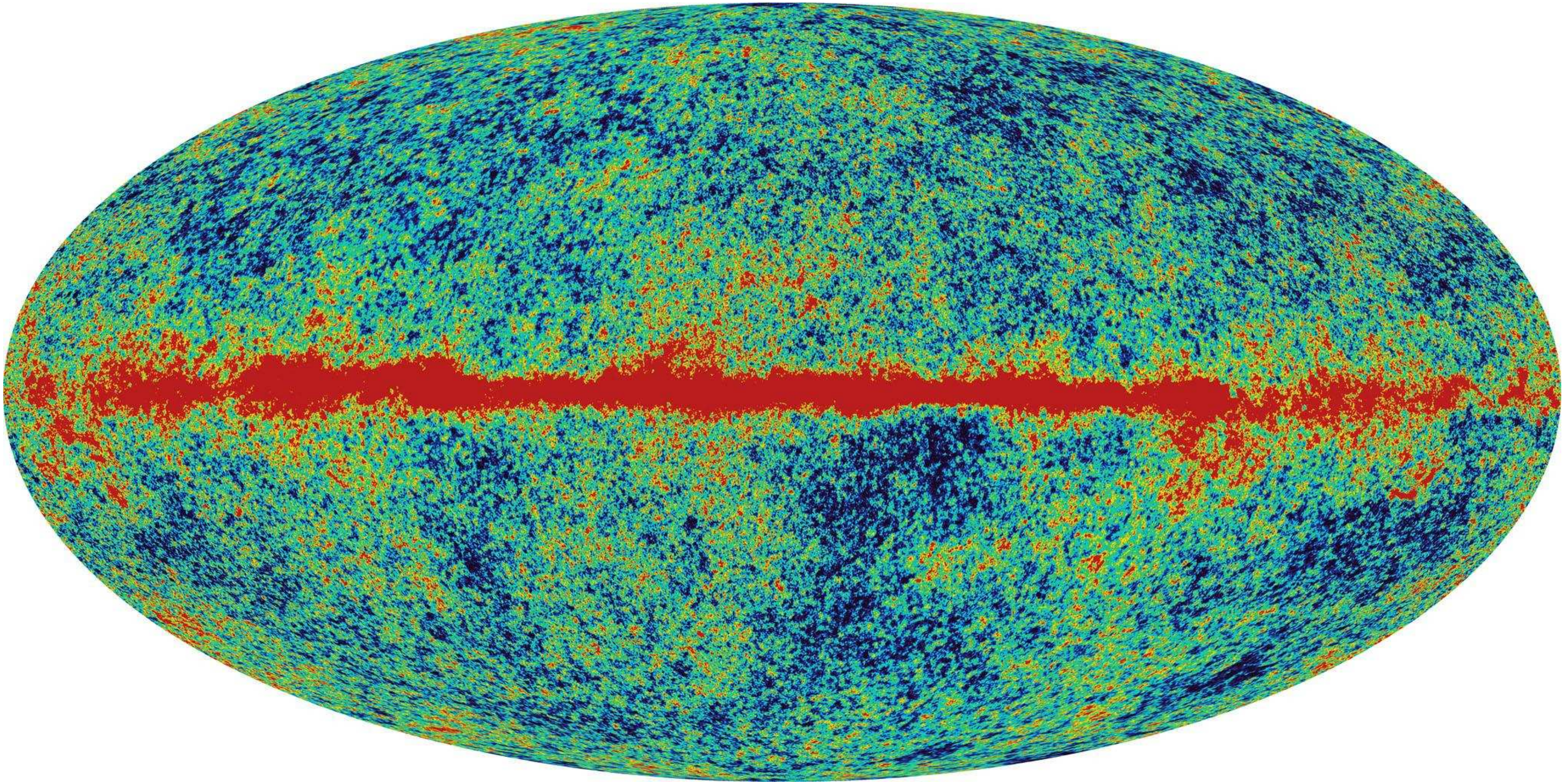
Sunyaev Zeldovich effect is expected to be strongest in Coma cluster, temperatures of -0.34 ± 0.18 mK in W and -0.24 ± 0.18 mK in K-band; barely detectable with WMAP, does not contaminate maps.



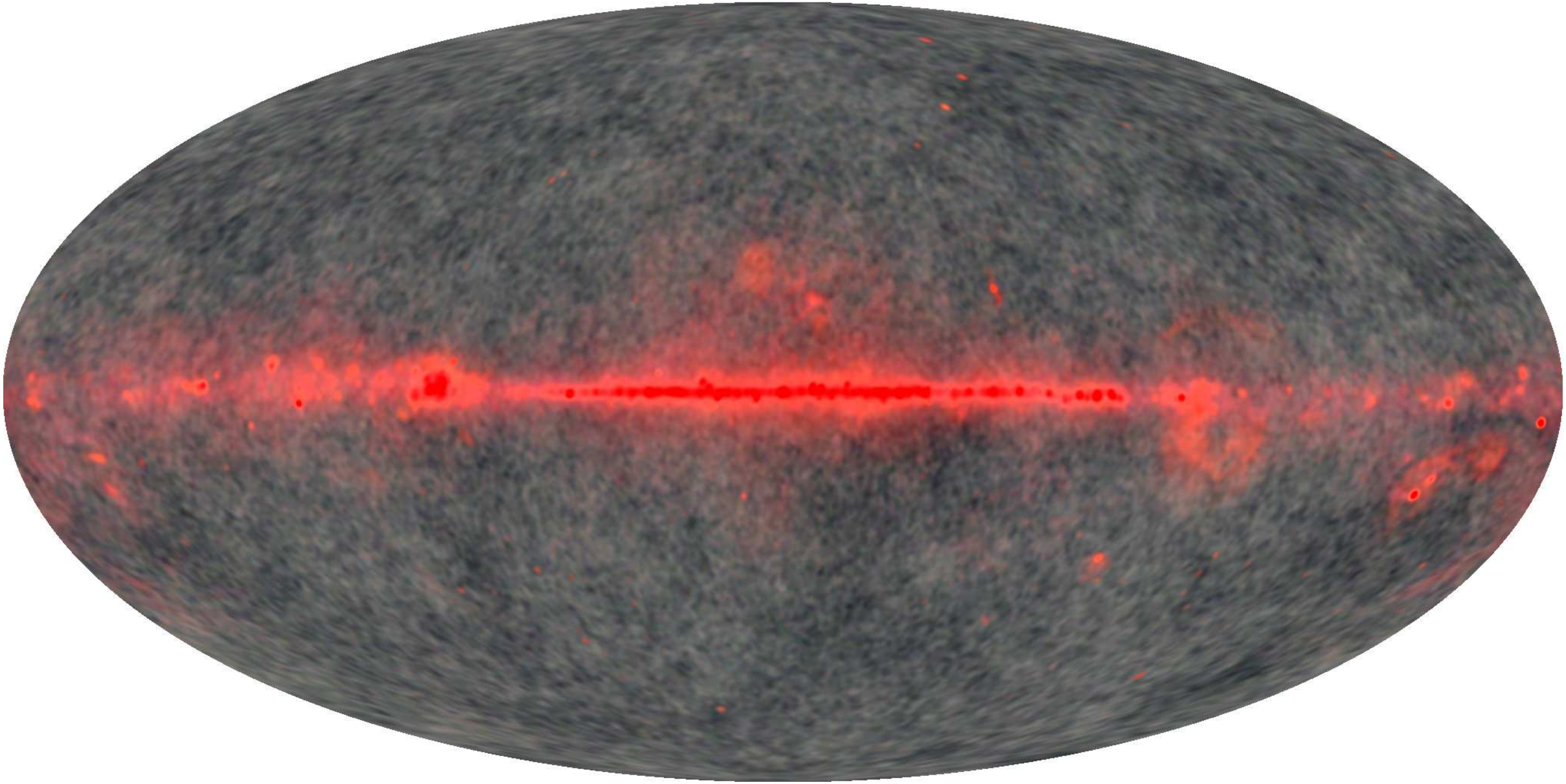
WMAP, K-Band, $\lambda = 13\,\text{mm}$, $\nu = 22.8\,\text{GHz}$, $\theta = 0.83^\circ$ FWHM



WMAP, Q-Band, $\lambda = 7.3\,\text{mm}$, $\nu = 40.7\,\text{GHz}$, $\theta = 0.49^\circ$ FWHM

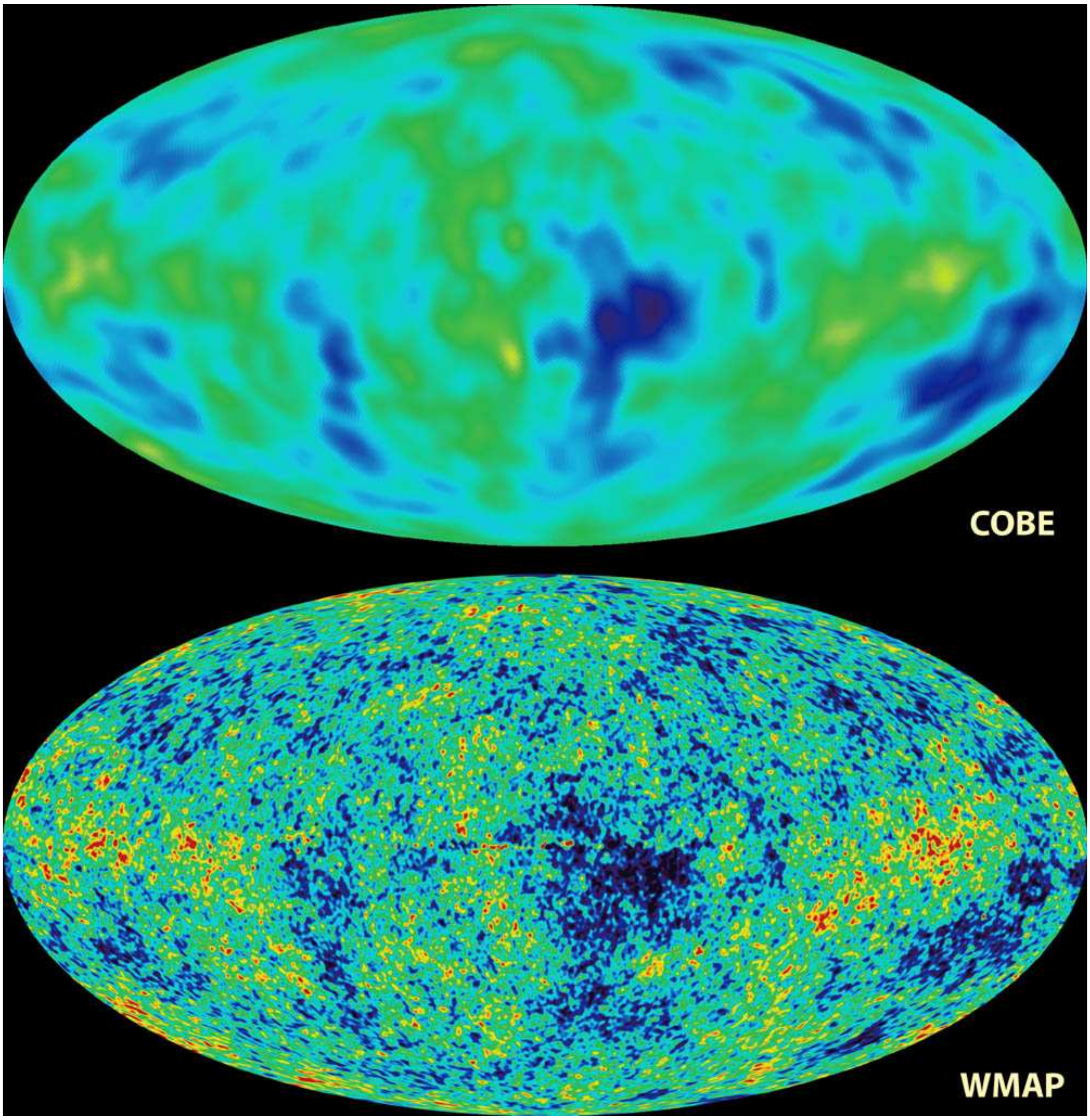


WMAP, W-Band, $\lambda = 3.2\,$mm, $\nu = 93.5\,$GHz, $\theta = 0.21^\circ$ FWHM



Different spectral signature enables identification of Galaxy foreground radiation

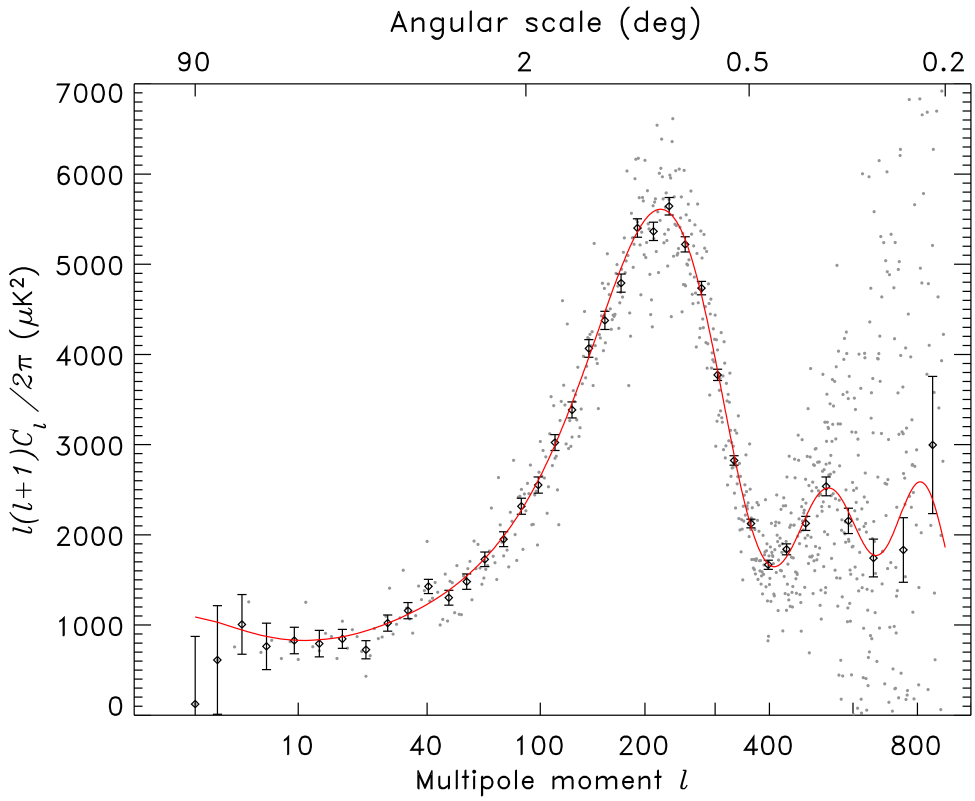
WMAP



After correction for foreground emission determine map of structure of the CMB.

WMAP data are best image of the CMB available

WMAP



(Spergel et al., 2003, Fig. 1)

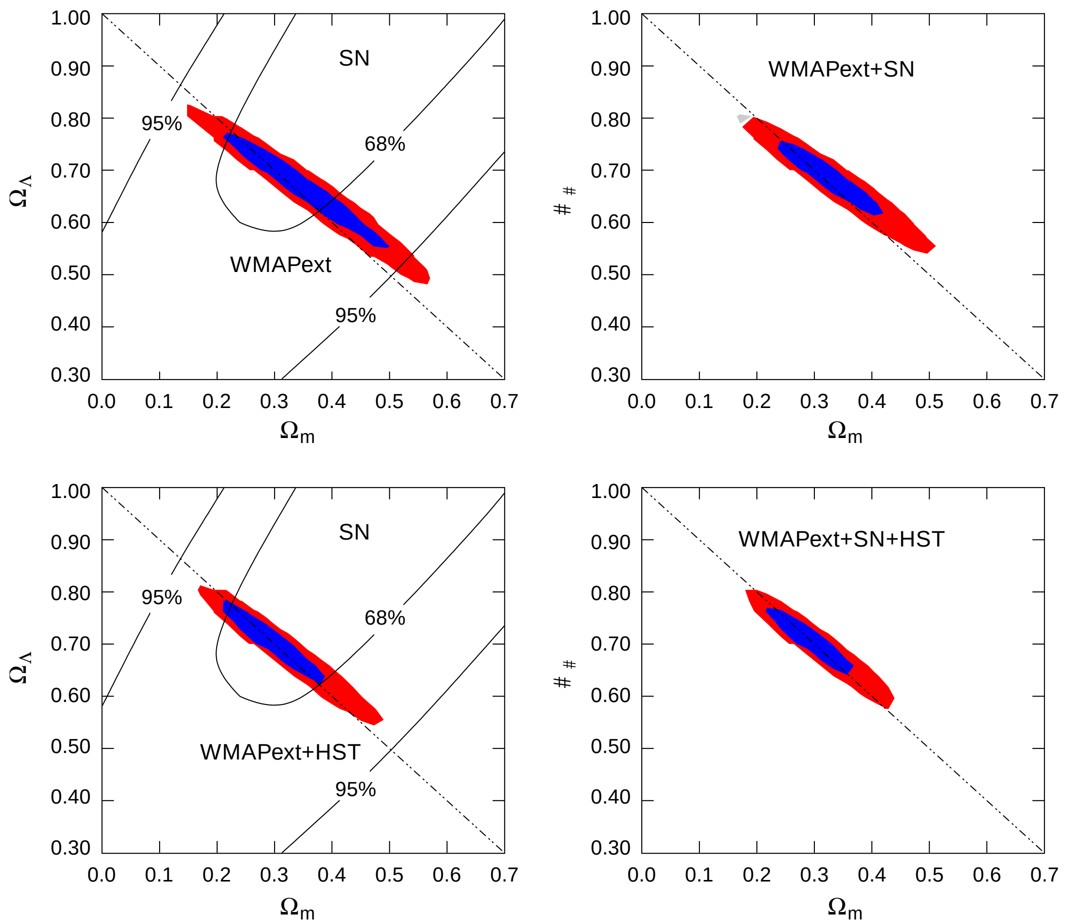
Best fit power-law ΛCDM to WMAP power spectrum $\Longrightarrow$ Very good agreement between data and theory

Best fit parameters for WMAP data:

$$
\begin{aligned}
h &= 0.72 \pm 0.05 \\
\Omega_{\mathrm{m}} h^2 &= 0.14 \pm 0.02 \\
\Omega_{\mathrm{b}} h^2 &= 0.024 \pm 0.01
\end{aligned}
\tag{8.86}
$$

(and assuming $\Omega = 1$)

WMAP



(Spergel et al., 2003, Fig. 13, SN contours are only given where they are not a prior in the analysis)

Removing constraint $\Omega = 1$

$\Longrightarrow$ Test how "flat" universe really is.

Using H_0 from HST and SN Iae results as priors into Bayesian analysis results in

$$\Omega = 1.02 \pm 0.02 \quad (1\sigma) \tag{8.87}$$

A model with $\Omega_\Lambda = 0$ is found to be consistent with the WMAP data only if $H_0 = 32.5\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ and $\Omega_{\mathrm{tot}} = 1.28$

$\Longrightarrow$ Ruled out by other measurements.

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The End